

Introduction: Identity

Robert Michels & Claudio Calosi

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Overview

1. The logic of identity
2. The Indiscernibility of Identicals and the Identity of Indiscernibles
3. Identity in the strict and loose sense
4. Identity, necessity, and determinacy
5. Generalized identity

Identity in first-order logic

- The standard formal theory of identity is classical first-order logic with identity (**CFOL**⁼)
- *Classical* means that the logic renders certain inference schemata and principles (including the laws of excluded middle and non-contradiction) valid and that its semantics is based on the idea that its formulas can only be either true or false (bivalent semantics)
- *First-order* that its predicates only apply to individual constants and variables and quantifiers only range over objects
- Classical first-order logic (**CFOL**) is *the* standard logic along with classical propositional logic – cf. any standard introduction to logic
- **CFOL**⁼ is a conservative extension of **CFOL**: additional logical symbol '=' is added to the formal language of first-order logic to extend its expressive power – that it's a *conservative extension* means that if we ignore all formulas involving the identity-sign, we have, logically and semantically, exactly **CFOL**

Identity in first-order logic

Two remarks about the language of $\mathbf{CFOL}^=$

- $\mathbf{CFOL}^=$ allows us to represent predicational (e.g. 'Anna is *gracious*.', 'Bernard *likes* Chris.') and quantificational (e.g. 'All *gracious* persons are *virtuous*.', 'There are people Bernard likes.') structure in language and state identity and distinctness claims ('Bernard is *identical* to Anna.', 'Anna is *distinct* from Chris.')
- Since $\mathbf{CFOL}^=$ treats identity as a logical notion, it treats intuitively valid inferences involving identity as *logically valid*; for example:
 1. Charles-Édouard Jeanneret is identical to Le Corbusier.
 2. Le Corbusier built the Cité radieuse.
 3. Therefore, Charles-Édouard Jeanneret built the Cité radieuse.
- If identity is treated as a regular, non-logical relational predicate (like e.g. 'x is next to y'), then such inferences would not count as logically valid (in classical first-order logic without identity)

Identity in first-order logic

Formal logic or formal ontology?

- Recall Varzi's point from the introductory lecture on formal ontology:

Arguably, the formal character of the identity relation is manifest. It is precisely because it is perfectly general and domain-independent that identity is often treated as a formal logical relation, given that formal logic is meant to yield absolutely general and domain-independent truths.[. . .] However, precisely because it is an objectual relation as opposed to a sentential operator—because it relates things in the world rather than truths about the world—I take this to be a reason to treat identity theory as part of formal ontology, not logic. (Varzi (2010), p. 5)

Identity in first-order logic

Syntax

Syntax of **CFOL⁼**

- The basis for any logic is the formal language in which it is formulated
- The syntax defines a grammar for the formal language by giving us a small set of *formation rules* which tell us exactly which sequences of *symbols of the alphabet* of the formal language are well-formed formulas, i.e. formulas which, so to say, make logical sense

Identity in first-order logic

Syntax: Alphabet

Alphabet of **CFOL**⁼

- Denumerably many predicates with a number of argument places indicated by an index: $F^1, G^2, H^3, \dots, F^n, G^n, H^n, \dots$
- Singular objectual variables: \dots, x, y, z
- Singular objectual constants: a, b, c, \dots
- Logical connectives: \neg, \wedge
- The universal quantifier: \forall
- Identity: $=$
- Parentheses to indicate quantifier scope and operator precedence: $(,)$

Identity in first-order logic

Syntax: well-formed formulas

Well-formed formula of **CFOL**⁼

- Atomic formulas:
 - For each predicate P^i with i argument places, for each sequence $\langle t_j, \dots, t_k \rangle$ which consists of i constants, variables, or terms of both kinds, $P^i t_j \dots t_k$ is a well-formed formula.
 - For each ordered pair $\langle t_j, t_k \rangle$ which consists of two constants, variables, or terms of both kinds, $t_j = t_k$ is a well-formed formula.
- Complex formulas:
 - If Φ and Ψ are well-formed formulas and v a variable, the following are well-formed formulas too:
 - $\neg\Phi, \Phi \wedge \Psi, \forall v\Phi$
- Nothing else is a well-formed formula.

The remaining standard connectives $\vee, \rightarrow, \leftrightarrow$ and the existential quantifier $\exists v$ are defined in the usual way in terms of negation, conjunction and the universal quantifier.¹

${}^1\Phi \vee \Psi =_{def} \neg(\neg\Phi \wedge \neg\Psi), \Phi \rightarrow \Psi =_{def} \neg(\Phi \wedge \neg\Psi),$
 $\Phi \leftrightarrow \Psi =_{def} (\Phi \rightarrow \Psi) \wedge (\Psi \rightarrow \Phi), \exists v\Phi =_{def} \neg\forall v\neg\Phi.$

Identity in first-order logic

Semantics

How is identity semantically characterized in **CFOL⁼**?

- Standard semantics for classical first-order logic: model-theoretic semantics
- Main idea: we use a set-theoretic structure called a model in order to define truth-conditions for formulas of **CFOL⁼**
- Model: A model \mathfrak{M} is an ordered set $\langle U, I \rangle$, where U is a set of object, the universe or domain of the model, and I is a function which maps constants to objects in U and predicates to sets of objects from U
- To interpret formulas containing quantifiers or free variables, we add an assignment ρ on the model \mathfrak{M} , which is a function from variables to objects in U
- Truth and falsity of a formula are evaluated relative to both a model and an assignment on the model

Identity in first-order logic

Semantics

Model-theoretic semantics – the intuitive idea

- The intuitive idea of model theoretic semantics is that we can use the set-theoretic machinery to assign *extensions* to variables, constants and predicates – variable \Rightarrow object, constant \Rightarrow object, predicate \Rightarrow set of objects (or set of tuples (ordered sets) in case of a relational predicate)
- Extensions are (roughly) the things to which these kinds of terms refer, i.e. an important aspect of their meaning
- In case of the identity relation, the extension is a set of ordered pairs containing each object in the domain of the model and itself
- The extensions of predicates and objects are used to model the relations of having a property (being an element of the extension of a predicate) and standing in a relation (being an element of an ordered set in a set of ordered sets in the extension of a relational predicate)

Identity in first-order logic

Semantics

Elements of a model

- The semantics for **CFOL**⁼ is given by a definition of truth for schematic sentences involving the non-logical and logical expressions of its language
- In the definition, we use the following elements:
 - $\mathfrak{M} = \langle U, I \rangle$ a model and ρ an assignment on \mathfrak{M}
 - t_0, t_1 two terms which are either constants, variables or one of each
 - The combined interpretation and assignment function $\rho I(t_k)$ which is $\rho(t_k)$ if t_k is a variable and $I(t_k)$ if t_k is a constant

Identity in first-order logic

Semantics: Definition of truth in a model relative to an assignment in **CFOL**

1. $P^i(t_1, \dots, t_n)$ is true in \mathfrak{M} relative to ρ iff $\langle \rho/I(t_1), \dots, \rho/I(t_n) \rangle \in I(P^i)$.
2. $\neg A$ is true in \mathfrak{M} relative to ρ iff A is false in \mathfrak{M} relative to ρ .
3. $A \wedge B$ is true in \mathfrak{M} relative to ρ iff A is true in \mathfrak{M} relative to ρ and B is true in \mathfrak{M} relative to ρ .
4. $\forall v(A)$ is true in \mathfrak{M} relative to ρ iff for every assignments ρ' on \mathfrak{M} which differ from ρ at most in v , A is true in \mathfrak{M} relative to ρ' .

The truth-conditions of all sentences involving other logical connectives and the existential quantifier derive from these clauses according to their definitions.

- The definition is recursive, which (for our purposes) means that we can use this small number of semantic clauses to evaluate all formulas, no matter how logically complex they are
- We now only need to add the clause for identity to complete the semantics for **CFOL⁼**

Identity in first-order logic

Semantics

What are the truth conditions for atomic identity formulas?

Truth in a model relative to an assignment can be defined as follows for atomic formulas involving = :

Truth for = A formula $t_0 = t_1$ is true in \mathfrak{M} relative to ρ if, and only if, $\rho I(t_0)$ is the same object as $\rho I(t_1)$.

Informally: A formula of the form $t_i = t_j$ is true iff the two constants/variables are assigned the same extension, i.e. the same object

Identity in first-order logic

Logical consequence and truth in $\mathbf{CFOL}^=$

- The model theoretic semantics for $\mathbf{CFOL}^=$ allows us to define the notions of logical consequence and of logical truth for that logic:

Logical consequence (in $\mathbf{CFOL}^=$) For any set of formulas Γ and formula Φ , $\Gamma \models \Phi$ (read: ' Φ is a logical consequence of Γ ') if, and only if, for all models \mathfrak{M} and all assignments ρ , if all formulas in Γ are true in \mathfrak{M} relative to ρ , then Φ is true in \mathfrak{M} relative to ρ .

Logical truth (in $\mathbf{CFOL}^=$) For any formula Φ , $\models \Phi$ (read: ' Φ is a logical truth') if, and only if, for all models \mathfrak{M} and all assignments ρ , Φ is true in \mathfrak{M} relative to ρ .

- (The logical truths are just the logical consequences of the empty set of formulas)

Identity in first-order logic

Identity as a relation

- Identity is treated as a logical connective in classical first-order logic, but the semantics also tells us something about identity conceived as a relation
- The extension assigned to the identity relation is the set of all ordered pairs which contain an object an itself
- So identity is the relation which holds between any object and itself and between nothing else
- Given this characterization, it follows that the identity relation as characterized by **CFOL**⁼ has certain formal properties:

Identity in first-order logic

Formal properties of the identity-relation

- Reflexivity: $\forall x(x = x)$
- Symmetry: $\forall x\forall y(x = y \rightarrow y = x)$
- Transitivity: $\forall x\forall y\forall z((x = y \wedge y = z) \rightarrow x = z)$
- (I.e. the identity relation is an *equivalence relation*)

These formulas are all logical truths of **CFOL**⁼, since they are true in every model relative to any assignment on the model

The Indiscernability of Identicals

A further, logical truth about identity (in **CFOL⁼**):

- Indiscernibility of identicals: $\forall x \forall y (x = y \rightarrow (Fx \leftrightarrow Fy))$

Why is it logically true? Intuitively, because assuming that x and y are identical, i.e. stand for the same (arbitrary) object, we cannot find any F which is such that one of x, y has it and the other lacks it. – Since x and y are the same, there are no extensions of predicates which contain one but not the other.

The Identity of Indiscernibles

Consider the following quote from Leibniz' correspondence with Clarke:

[T]here are not in nature two real, absolute beings, indiscernible from each other; because if there were, God and nature would act without reason, in ordering the one otherwise than the other; and therefore God does not produce two pieces of matter perfectly equal and alike. (Leibniz et al. (2000), L V.21)

Leibniz here argues (using the Principle of Sufficient reason) that in nature, there are no two things which are indiscernible, or to put it differently, that *any two indiscernible things are identical*.

Similarly, Quine accepts a discourse-relative version of the same principle:

Objects indistinguishable from from one another within the terms of of a given discourse should be construed as identical for that discourse. (Quine (1950), p. 626.)

Is this principle (discourse-independently) also a logical truth?

The Identity of Indiscernibles

- Identity of indiscernibles (Leibniz' Law):

$$\forall x \forall y ((Fx \leftrightarrow Fy) \rightarrow x = y)$$

Unlike the Indiscernibility of Identicals, this principle is not a logical truth of **CFOL**⁼

The Identity of Indiscernibles

Why Identity of Indiscernibles (LL) is not a logical truth

- LL tells us that for any x, y which do not differ regarding whether they have F , the x, y are the same object
- The problem is that even if x, y do not differ with respect to F , they can still differ with respect to a different property G
- And if they do, x, y are really distinct by the Indiscernibility of Identicals

The Identity of Indiscernibles

Verifying LL by going second-order?

- Why is LL not a logical truth of **CFOL**=?
- Perhaps because this logic lacks the means to quantify over properties – the idea is that if the principle said in its antecedent that x, y are indiscernible regarding *all properties* and not just an arbitrary property F , then it should be a logical truth

The Identity of Indiscernibles

- Classical second-order logic with identity provides the means to formulate this suggested version of the principle:

Identity of indiscernibles – second-order version LL2

$$\forall x \forall y (\forall F (Fx \leftrightarrow Fy) \rightarrow x = y)$$

- Is LL2 a truth of classical second-order logic with identity?
- There is not one standard semantics for second-order logic as in first-order logic; rather there are two different candidate semantics
 - *Full semantics*, which is based on the idea that every set of objects from the domain of a model corresponds to a property
 - *Henkin semantics*, which restricts the range of models from the full semantics to those which satisfy all (previously determined) axioms of second-order logic, including in particular the Comprehension axiom schema and the Axioms of choice

The Identity of Indiscernibles

- In the full semantics, LL2 is a theorem
- Explanation: for any objects a and b in a model, we will have the sets $\{a\}$ and $\{b\}$ as semantic values of predicates in the model, i.e. the model will tell us that there are properties which are only had by a and by b respectively
- This means that the antecedent of the principle is false, since there are no two objects which are indiscernible (the properties which only they have always discern them); so the whole principle is trivially true (since any conditional with a false antecedent is true)
- For the same reason, LL2 is also true in the Henkin semantics

The Identity of Indiscernibles

LL2, second-order logic, and metaphysics

- LL2 is not a theorem of classical first-order logic with identity, but a theorem of second-order logic
- There is a big discussion in metaphysics about whether LL2 is true; see the paper by F. A. Muller to be discussed next week!
- Is this discussion void in light of the logical status of LL2, which is arguably closer to what we want the principle to say than LL (the first-order version)?

The Identity of Indiscernibles

LL2, second-order logic, and metaphysics

- One might argue that LL2 still falls short of expressing the principle about identity which we are interested in
- The universal quantifier in LL2 ranges over absolutely all properties/relations, including the trivializing properties had only by one particular object (i.e. properties like 'being *a*') whose presence in a model guarantee the truth of LL2
- Perhaps what the principle is really supposed to say is that any objects *which are indistinguishable with respect to all non-trivializing properties* are identical
- Such a restricted version of LL2 is not logically true in the Henkin semantics, since we can construct models which falsify it (e.g. a model with two distinct objects *a* and *b* in its domain which are both only in the extension $\{a, b\}$ (or simply in no extension at all), making them indiscernible, but distinct in that model) – so logic might leave room for disagreement about the principle after all

Which notion of identity is captured by identity in **CFOL**=?

- According to **CFOL**⁼, identity is a transitive relation
- But is this plausible?
- Consider the following piece of reasoning:

17 years ago, Mel as a one year old had no teeth, but now Mel as an 18 year old has 32 teeth. Since Mel is still the same person, Mel as a one year old and Mel as an 18 year old are identical. But by the Indiscernibility of Identicals (a logical truth!), if Mel as a one year old and Mel as an 18 year old are identical, then for an arbitrary F , the two are indistinguishable with respect to F . But they aren't with respect to having teeth!

Which notion of identity is captured by identity in **CFOL**=?

- Is something wrong with the notion of identity characterized by **CFOL**=?
- No, it just captures a very strict, in fact the strictest notion of identity, *numerical identity*
- Examples like the Mel-example have moved philosophers since at least Locke, to distinguish between identity in this sense and *identity in the loose sense*
- Numerical identity is a matter of sharing all properties, but objects which differ in some properties can still be identical in the loose sense
- Given this distinction, we may say that personal identity (including that of Mel) is identity in the loose sense, whereas **CFOL**= characterizes the notion of numerical identity between objects, i.e. the strict(-est) notion

Personal identity as loose identity

- Important question about loose identity: under which conditions are numerically distinct things loosely identical?
- This question is at the core of the discussion of personal identity
- What makes the two Mel-stages in the example *the same person*?
- Some answers proposed in the literature (cf. Olson (2019) for references):
 - *Psychological continuity*: the later Mel-stage is the same person as the earlier Mel-stage because it inherits some important mental features, such as e.g. beliefs, memories, preferences, etc. (Locke, Parfit)
 - *Animalism*: the two Mel-stages are the same biological organism (van Inwagen, Olson)
 - *Sameness of soul*: the two Mel-stages have the same soul (Plato, Descartes, Swinburne)

Processes and time

Heraclitus ever-changing waters

- Another category of entities which persist (and change) through time are processes
- Consider Quine's discussion of Heraclitus claim that you cannot bathe in the same river twice, 'for new waters are ever flowing in upon you' (Quine (1950), p. 621.)

[...] you can bathe in the same river twice, but not in the same river-stages. You can bathe in two river-stages which are stages of the same river, and this is what constitutes bathing in the same river twice. A river is a process through time, and the river-stages are its momentary part. Identification of the river bathed in once with the river bathed in again is just what determines our subject-matter to be a river process as opposed to a river stage. (ibid.)

Processes and time

Heraclitus ever changing waters

- River stages are numerically distinct temporal snapshots of a river, temporal parts (compare the introduction to mereology)
- They stand in the relation of 'river-kinship', a relation of loose identity
- The river, a process which extends through time, consists of all its temporal parts and two rivers are numerically identical if their temporal parts are

Is loose identity identity?

Identity is utterly simple and unproblematic. Everything is identical to itself; nothing is ever identical to anything else except itself. There is never any problem about what makes something identical to itself; nothing can ever fail to be. And there is never any problem about what makes two things identical; two things never can be identical. [...] We do state plenty of genuine problems in terms of identity. But we needn't state them so. Therefore they are not problems about identity. (Lewis (1986), pp. 192-193.)

If Lewis is right, we shouldn't think of loose identity as a genuine alternative kind of identity, but rather focus on the relevant 'replacement-relations' like river-kinship or psychological continuity and the condition under which they obtain.

The necessity of identity

- An important question about identity: is identity a necessary relation?
- If Mark Twain is Samuel Clemens, is this necessarily the case?
- There is a simple proof in first-order modal logic for the necessity of identity which is often attributed to Kripke, who presented it in Kripke (1971)
- Kripke was however not the first to find the proof; see Burgess (2013) for the story involving Wiggins, Barcan Marcus, and Quine
- In the following, we'll use the symbol ' \Box ' for the necessity operator (as is standard), so that ' $\Box\Phi$ ' says that the formula ' Φ ' expresses a necessity

The necessity of identity

The proof

1. $\forall x \Box(x = x)$ ('Every x is necessarily identical to x ')
 2. $\forall x \forall y (x = y \rightarrow (\Box x = x \rightarrow \Box x = y))$ ('Every x and y are such that, if the two are identical, then if it is necessary that x is x , then it is necessary that x is y ')
 3. $\forall x \forall y (x = y \rightarrow \Box x = y)$ ('Every x and y are such that, if they are identical, they are necessarily identical')
- The argument is logically valid and appears to also be sound, i.e. to have true premises

The necessity of identity

Truth of the premises

1. $\forall x \Box(x = x)$
 2. $\forall x \forall y (x = y \rightarrow (\Box x = x \rightarrow \Box x = y))$
 3. $\forall x \forall y (x = y \rightarrow \Box x = y)$
- 1. is postulated to be true (see a later remark)
 - 2. is an instance of an axiom schema corresponding to Indiscernibility of identicals:
 - $\forall x \forall y (x = y \rightarrow (\Phi(x/z) \rightarrow \Phi(y/z)))$
 - where $\Phi(v/z)$ is any formula involving free (i.e. not bound by a quantifier) occurrences of z which have all been replaced by the variable v (i.e. in the axiom schema, x in the first and y in the second occurrence of the schematic formula)
 - The particular formula needed to generate 2. is $\Box x = z$; instances of axiom schemata are logically true, so 2. is true

The necessity of identity

Validity

1. $\forall x \Box(x = x)$
2. $\forall x \forall y (x = y \rightarrow (\Box x = x \rightarrow \Box x = y))$
3. $\forall x \forall y (x = y \rightarrow \Box x = y)$

- 3., the conclusion follows from 1. and 2. since:
- 1. renders the formula in the antecedent of the embedded conditional in 2. true
- This makes the whole embedded conditional $(\Box x = x \rightarrow \Box x = y)$ equivalent to its consequent $(\Box x = y)$ for all values for x and y
- This in turn means that assuming that 1. is true, 2. is equivalent to, and hence implies, 3.

The necessity of identity

Not quite so innocent and not so easy to make use of (see Burgess (2013))

- Kripke in fact only postulates 1. conditional on two subtle, but significant assumptions about the necessity operator, namely that it expresses *metaphysical necessity* and *weak necessity*
- There is no direct way to use to proof to justify the inference from $a = b$ to $\Box a = b$, i.e. the necessity of identity for identity claims involving constants, arguably the sort of inference which is most interesting for metaphysicians

The necessity of identity

Pushback against the necessity of identity

- Not all philosophers accept the necessity of identity
- Gibbard has argued for a view which allows contingent identity, e.g. between a statue and the lump of clay it is made from
- The idea is the statue and the lump have different persistence criteria: it is possible for the lump to persist through a loss of its shape, but not for the statue (see Gibbard (1975); for general discussion see Schwarz (2013))

Identity and indeterminacy

- Are there indeterminate identities, i.e. identity claims which are neither determinately true, nor determinately false?
- An infamous argument in Evans (1978) which was supposed to show that there are no indeterminate objects, but which instead shows that if it is indeterminate whether $a = b$, then a and b are distinct
- Here is the argument:

Identity and indeterminacy

Evan's argument:

The symbol ' ∇ ' is here used to express indeterminacy, as in: ' $\nabla\Phi$ ' – saying that it is indeterminate whether Φ

1. $\nabla(a = b)$ ('It is indeterminate whether a is identical to b .')
2. $\lambda x[\nabla x = a](b)$ (' b has the property of being indeterminately identical to a .')
3. $\neg\nabla(a = a)$ ('It is not indeterminate whether a is identical to a .')
4. $\neg\lambda x[\nabla x = a](a)$ (' a does not have the property of being indeterminately identical to a .')
5. $\neg(a = b)$ (' a is distinct from b .')

1. entails 2.; 3. is an additional assumption which entails 4.; 5. follows from 2. and 4. via the contraposition of Indiscernibility of Identicals:

$$\forall x\forall y(\neg(Fx \leftrightarrow Fy) \rightarrow \neg x = y)$$

Identity and indeterminacy

Importance of Evans's argument

- Evans's argument (which is published in a one page paper!) has kicked off a discussion of whether there is metaphysical indeterminacy, indeterminacy in the world as opposed to in our language describing it, which lasts to this day
- There are numerous discussions and responses to the argument, e.g. Lewis (1988), Parsons and Woodruff (1995), Barnes (2009)

Generalized identity

Generalized identity

- Syntactically, the generalized identity operator ' \equiv ' two open (containing free variables) or closed (containing no free variables) sentences and optionally, one or more variables as a subscript, to form a sentence
- Using the operator we can for example express:
 - $p \equiv q$
(‘For it to be the case that p is for it to be the case that q .’)
 - $F_x \equiv_x G_x$
(‘For a thing to be F is for it to be G .’)

Generalized identity

Generalized identity

- What makes generalized identity interesting are its connections to other important metaphysical notions, such as essence and grounding – it seems that we can use generalized identity to characterize or perhaps even reductively define these notions
- Consider essence – metaphysicians like to make claims like: 'Sets essentially have their elements.', 'Souls are essentially abstract.', or 'Coloured objects are essentially in spatiotemporally extended.'
- These claims are claims of *partial generic essence*, they specify *part* of what it is to be e.g. a coloured object (generic, i.e. it's not the essence of a particular object, but rather of a certain quality)

Generalized identity

Generalized identity

- According to Correia and Skiles (2017), we can characterize partial generic essence as follows:
 - $Gx \equiv_x Fx \wedge Hx$
- Informally, stated *being F* is partially what it is *to be G* if, and only if, there is some *H* such that for a thing to be *G* is for it to be both *F* and *H*
- Since the definition crucially relies on a conjunction, we can say that this notion of essence is defined in terms of the notion of a *conjunctive part*
- E.g. to be rational and to be an animal are conjunctive parts of being human – the essence of being human includes being an animal and being rational)

Final words

Some other topics not covered in this presentation (References on request!)

- Identity in Quantum Mechanics (French and Krause)
- Can we do without identity? (Wittgenstein, Wehmeier)
- What are the truthmakers for identity-claims?
- Is identity fundamental? (Hochberg, Mantegani)
- Are there identity-criteria objects which have to fulfil in order to count as identical?
- Is identity relative (e.g. to a sortal concept, to times,...)? (Geach)
- Are objects identical to their parts? (Baxter)
- What is the relation between identity and identification?



Thank you!

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