Preprint; final version forthcoming in *Metaphysics*

Multidimensional Properties: Primitivism vs Reductionism

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September 23, 2025

The focus of this paper is on the metaphysical nature of multidimensional properties, properties like virtue, beauty, or health which can be had in different ways and to different degrees. Do such properties form their own sui generis kind, as existing discussions seem to suggest, or can we rather reductively account for them, relying on notions which may already be present in our ontology or ideology? In this paper, I will introduce a reductionist account of multidimensional properties, making a partial case for Reductionism.

1 Introduction

Imagine three friends, Sam, Max, and Kim. Sam has a rich and diverse body of knowledge, but is not very good at taking fair decisions. Max on the other hand is very good at taking fair decisions, but doesn't have the same amount of knowledge Sam has. Kim has both of the traits of which Sam and Max each have and lack one.

Having rich and diverse knowledge and taking fair decisions are what one might call 'wisdom markers': a person who has at least one of them can under certain circumstances be rightfully called wise. This includes Sam, Max, and Kim in our little example.

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However, just attributing wisdom *simpliciter* to the three friends would leave three important aspects of the situation unexplained. First, while it seems right to call Sam, Max, and Kim wise, the three are wise in distinct ways. Sam is wise in a particular way in which Max is not, and vice versa, and Kim in turn is wise in both mentioned ways. Second, assuming for the sake of illustrations that *having a rich and diverse body of knowledge* and *being able to take fair decisions* count differently towards being wise and add up if had together by the same person, Sam, Max, and Kim can also rightly be said to be wise to different degrees. Furthermore, if we assume that the previous description exhausts the ways in which they are wise, there may be situations in which Sam and Max, and plausibly also Kim, count as somewhat wise, but still fall short of being wise simpliciter. Think e.g. of a situation in which we compare them to a paradigm case of a wise person, such as Aristotle.

From the metaphysical perspective which I assume in this paper, examples of this kind call for an explanation of four things. First, of the overall similarity between things which share a certain feature F, but also of two kinds of difference they may nonetheless exhibit with respect to that same feature. Namely a qualitative difference with respect to the different ways in which things can be F, as well as a quantitative difference related to the different degrees to which things can be F. Lastly, the apparent fact that there are some situation in which something has the feature in question, but still is not F simpliciter, should also be explained.

A rather straightforward way to provide such a four-partite explanation builds on the idea that there are particular complex properties, *multidimensional properties* (see van Woudenberg and Peels (2018), Calosi and Michels (2025)). Applied to the example: Sam, Max, and Kim overall resemble each other since they have the same multidimensional property, namely *wisdom*. Still, they are qualitatively different with respect to wisdom since they have different (sets of) *aspects* of this multidimensional property, where an aspect is a property which, in a way to be specified, is conducive to having that multidimensional property. The three friends are furthermore quantitatively different with respect to wisdom, since they differ not only in which, but also in how many aspects of this property they have, where the particular aspects at issue may contribute to larger or smaller degrees to being overall wise. Finally, there may be situations in which the three may fall short of being wise simpliciter, since they have some, but not all aspects of wisdom needed to qualify as wise simpliciter in the relevant situation.

This metaphysical elaboration of the example naturally aligns with standard positions in the literature on wisdom. It is in particular commonly assumed that wisdom comprises practical and theoretical wisdom, giving us already two qualitatively different ways of

being wise, which in turn comprise different ways of being practically and theoretically wise (see Swartwood and Tiberius (2019), p. 19). Furthermore, there is a large literature in psychology and philosophy investigating the components of wisdom (see ibid, pp. 25ff) and there are different proposals in psychology for how wisdom can be measured (see e.g. Kunzmann (2019)).

A further notion which exhibits a similar structure is that of health (in the sense of being a healthy human organism). As in the case of wisdom, the claim that health is a multidimensional property goes beyond claims usually made in the contemporary analytic literature, which is generally neutral regarding the metaphysical category of health. That health has a multidimensional structure is however backed by all three of the current standard approaches to defining the notion.[1] According to the naturalist approach (see Boorse (1977, 1997)), an organism is healthy if, considered relative to a reference group to which it belongs, none of the normal (in the statistical sense) functions of its parts or inherent processes are impaired. If we assume that health is a multidimensional property, that the functioning of an organism is unimpaired with respect to one such part or inherent process may be taken to indicate that it has a particular aspect of health. The two other standard approaches to defining the notion, the normative and the hybrid approach, likewise attribute a multidimensional structure to health. (Cf. Ereshefsky (2009).) Normative theories define health in terms of meeting the (arguably multiple) standards or norms which determine which states of organisms society considers desirable. Hybrid theories combine elements of naturalist and normative theories, which, as in the case of wisdom, already gives us a top-level distinction between what one can take to be two aspects of health, each of which in turn exhibits a multidimensional structure in itself.

Wisdom and health are just two among a large number of notions which exhibit the same sort of multidimensional structure which, from a metaphysical perspective, calls for the same kind of explanation. To name just a few further examples drawn from the recent literature: (artificial) agency (see Dung (2024), §2), belief (see van Woudenberg and Peels (2018), §8), creativity (see Calosi and Michels (2025), §5.2), freedom (see van Woudenberg and Peels (2018), §7), naturalness (of conventions) (Gasparri (2024)), rationality (Siscoe (2024)), and values (Hedden and Muñoz (2023)), have all been suggested to be multidimensional in some way. There are more or less extensive philosophical

[1] Note that there are uses of the words 'health' and 'healthy' which fall outside the scope of these definition. Think e.g. of the claim that someone has a healthy sense of their own aptitudes, or that celery is healthy. What I say in the following does not commit me to the claim that all such claims point to the same multidimensional property. I thank an anonymous referee for prompting me to clarify this.

discussions of all of these notions. However, general metaphysical questions about them, such as those raised above, have not yet received much attention in the literature, with two notable exceptions.

First, van Woudenberg and Peels (2018) discuss the idea that certain degree-involving sentences imply the existence of multidimensional properties. Their discussion is important, since it provides what appears to be the first characterisation of such properties in the recent literature and since it partly makes the case that they should be taken to belong to a category of their own. The latter is done by arguing that they differ from more familiar kinds of properties, including determinable properties, and that the relation between them and their aspects is neither that between types and tokens, nor that between genus and species.

Second, Calosi and Michels (2025) argue that multidimensional properties can be used to make sense of the idea that qualities can be had to intermediate degrees and propose that such properties have different modes of having. Their paper provides a slightly more elaborate account of multidimensional properties, but still does not address certain important metaphysical questions.

In this paper, I want to focus on two such questions. The first question is that of the nature of the relation between multidimensional properties and their aspects (call this the *aspecthood question*). Let me briefly explain the significance of this question and how I will approach addressing it in this paper.

My introduction of multidimensional properties draws a certain picture of what multidimensional properties are. For all I have said, they are complex properties, and their complex intrinsic structure in some sense involves their aspects, which themselves are properties. I like this picture, but it involves a number of explicit and implicit metaphysical commitments which not everyone might be comfortable with, and, more perspicuously, which, at this point in this paper, one need not necessarily make. The underlying idea, that we can understand the nature of multidimensional properties by understanding their intrinsic structure, might hence be seen as contentious. To not prejudice the discussion, I will hence not frame the aspecthood question in these terms.

Instead, I will assume that what any account of multidimensional properties needs to explain is a characteristic pattern of necessary co-instantiation. Namely, to have a multidimensional property, a thing also has to have at least one of its aspects. I will assume in the following that this is a basic datum which any answer to the aspecthood question has to explain, making the instantiation of this pattern a necessary condition for being a multidimensional property. A good answer to the question hence illuminates the nature of the relation between multidimensional properties and their aspects in the

sense that it delivers a ground or metaphysical explanation of this pattern of necessary co-instantiation. The answer I will propose later on in the paper relies on the intrinsic structure of multidimensional properties, but one could also think of an answer which draws e.g. only on extrinsic relations between wholly distinct properties, or which simply characterizes the relation axiomatically without committing to any sort of metaphysical claim about the nature of the relation.[2]

The second question is whether we can provide a reductive theory which explains the apparent multidimensional structure exhibited by prototypical examples of what appear to be multidimensional properties in terms of more familiar notions (call this the *reduction question*). Let me make more explicit what I mean by 'reductive' in this context. As previously stated, multidimensional properties play certain theoretical roles, including that they allow us to explain certain qualitative similarities between things, that they provide us with a way to account for the qualitative and quantitative differences which still obtain between things which are similar in these ways, and that they allow us to account for the apparent fact that not everything exemplifies the relevant quality to a sufficient degree to count as having it simpliciter in all contexts.

I will call a theory *reductive* if it provides a metaphysical inventory of notions which allow one to account for these roles without positing a primitive distinction between multidimensional and non-multidimensional properties as part of its ideology.[3] An answer to the *aspecthood question* may point into the direction of an answer to the *reduction question*: If we can wholly explain the theoretical roles apparently played by multidimensional properties in terms of more familiar metaphysical notions, then we do not need to assume a primitive distinction between multidimensional and non-multidimensional properties. This strongly suggests a positive answer to the reduction question (the appar-

[2]Note that an account which answers the aspecthood question does not necessarily give us a definition (in the sense of a list of necessary conditions which together are sufficient for an object to belong to the defined category) of what makes a property multidimensional. An account which in addition meets the explanatory demands may come closer, since I take these demands to spell out part of a theoretical role which multidimensional properties can play, but still doesn't get us there. My view is that progress towards a better understanding of the metaphysics of multidimensional properties can be made without providing, or departing from a definition of this sort. As is the case with many other metaphysical notions, a sensible discussion can also be had if one has a robust list of paradigmatic examples to work with.

[3]By 'ideology', I mean any kind of metaphysical posit which goes beyond what counts as an ontological commitment of a theory according to the classical Quinean criterion. So a reductive theory may rely e.g. on different distinctions between properties, intentional or hyper-intentional notions, etc.. Note also that this conception of reduction is neutral regarding whether multidimensional properties exist. For all it tells us, a reductive theory of multidimensional properties might either insist that there are no multidimensional properties, or instead say that they are identical to the (existing) surrogate entities the theory posits to play their theoretical roles (and thus exist). I will leave open what the best way for a Reductionist to go is.

ent paradigm examples of multidimensional properties and the associated roles captured by the four explanatory demands can be accounted for in terms of other metaphysical notions). I will introduce and elaborate a specific take on Reductionism in section 3. In contrast, if some of the roles apparently played by multidimensional properties cannot be accounted for by the surrogate entities posited by our metaphysical theory, then we do have to make this assumption. This points to a negative answer to the Reduction question (multidimensional properties form a non-reducible, sui generis kind). There are however also possible answers to the aspecthood questions which do not prejudice the reduction question. Think e.g. of a formal theory which describes the relation between multidimensional properties and their aspects in terms of a set of axioms which fix the logical relations between sentences involving certain predicates.

Neither van Woudenberg and Peels (2018), nor Calosi and Michels (2025) conclusively answer either of the two questions, though the arguments distinguishing multidimensional from other kinds of complex properties provided by van Woudenberg and Peels at least point towards Primitivism.[4]

This paper contributes to the project of fleshing out the metaphysics of multidimensional properties by investigating these two questions. I will in particular argue that given van Woudenberg and Peels' characterization of multidimensional properties, a metaphysical reduction of the aspecthood-relation in terms of another kind of complex properties is available. This answers the *aspecthood question* and also strongly suggests a Reductionist answer to the *reduction question*.

However, it turns out that the resulting account is not satisfactory, in that it does not fully address the explanatory demands set out earlier in this section. To address this problem, I then suggests a way to generalize this account to Calosi and Michels (2025)'s augmented account of multidimensional properties, which incorporates different modes of having such properties, lending further support to a Reductionist view of multidimensional properties.[5]

[4]Note that both of these questions should be distinguished from the distinct metaphysical question of whether the multidimensional property is metaphysically prior to its aspects, or vice-versa. The distinction between Primitivism and Reductionism is orthogonal to these two views and while an answer to the *aspecthood question* may in some cases suggest a particular answer to this third question, it need not necessarily do so. I will not further discuss this in this paper and mention it only to avoid confusions.

[5]It should already be evident that this paper is written in a particular realist spirit: Throughout the paper, I assume that when we talk about health, wisdom, virtue, etc.. these words can be taken to pick out something in our ideology/ontology which accounts for the qualitative nature of particulars, namely properties. Note that the realist approach may be thought to be controversial for at least two reasons. First, it obviously conflicts with Nominalist approaches to metaphysics. Second, the focus on properties ignores realist alternatives which e.g. posit tropes instead of properties. In the context of this paper, I will neither be able to defend property-based Realism against Nominalism,

The plan of the paper is as follows. In the next section (§2), I will introduce van Woudenberg and Peels' account of multidimensional properties and slightly expand on the negative part of their characterisation by providing arguments against three further reductive accounts of aspecthood. I will then argue that given this account, multidimensional properties can be metaphysically reduced to what I will call conjunctive-disjunctive properties, where the aspecthood-relation can within this account be perspicuously described using Correia and Skiles (2019)'s notion of quasi-subsumption (§3). In §4, I will introduce Calosi and Michels' distinction between four modes of having a multidimensional property. I will then first argue that without them, the proposed Reductionist account cannot easily meet the explanatory demands I introduced earlier in the paper and provide a solution to the resulting problem in the form of variants of the definitions from Calosi and Michels (2025) which are adapted to the Reductive account. The last section (§5) wraps up the discussion and points to a subtle difference between Reductionist and Primitivist accounts, which may matter for further developments of the accounts.

2 van Woudenberg and Peels' Complex, Resultant Properties

Positing multidimensional properties allows us to provide an answer to the metaphysical questions about the nature of health, wisdom, freedom, and so on, raised in the introduction. These answers however remain incomplete if not paired with an account of the metaphysical nature of such properties. What does such an account have to bring to the table? Crucially, it has to explain whether multidimensional properties are a further addition to our ideology, forming their own, sui generis kind (Primitivism), or whether they are really reducible to either other kinds of properties or bits of metaphysical ideology which one may already have in one's ideological inventory (Reductionism). But in order to discuss these two opposing positions, we first need a clear idea of what it takes for a property to count as multidimensional, no matter whether multidimensionality is ultimately reducible or not. A characterization of this sort need not tell the full meta-

nor will I be able to compare it to Realists alternatives. Still, it is important to point out that Realist approaches to the particular qualities discussed in this paper are live options, judging by the philosophical literature about some apparently multidimensional notions. E.g. in the case of *virtue*, think of recent non-naturalist versions of moral realism; see e.g. Shafer-Landau (2003), §3, Sayre-McCord (2023), §2. Note furthermore that due to the focus on metaphysical questions, I will also not be able to relate the metaphysical views discussed in this paper to important work on multidimensionality in language. Multidimensional adjectives are e.g. discussed in Sassoon (2013), Grinsell (2012) discusses multidimensional vagueness, and D'Ambrosio and Hedden (2023) have very recently contributed an insightful philosophical take on this discussion. These task will have to wait for another occasion.

physical story, but it certainly has to be detailed enough to enable us to identify typical examples of multidimensional properties as such.

van Woudenberg and Peels (2018) provide a characterisation which meets this requirement. It identifies a substantial theoretical role multidimensional properties can play, makes some progress towards answering the *aspecthood question*, and dismisses some reductionist accounts which metaphysically explain this structure in other, more familiar terms. This latter aspect of their characterisation at least suggests, though does not fully establish, a Primitivist account.

Before I introduce their account, a terminological remark has to be made. van Woudenberg and Peels call the properties they posit 'complex, resultant properties' and instead of 'aspect', they sometimes use the term 'elementary property'. I take this to be a terminological difference and will hence continue relying on the terminology used in Calosi and Michels (2025) in presenting their account.

The main focus of van Woudenberg and Peels (2018) is on two metaphysical questions about degree-involving sentences, namely whether they imply that (A) there is something which is degree sensitive and (B) that there is more or less of that something. Their main point is that for certain such sentences (two of their examples are: (4) 'Socrates is wiser than Diogenes.' and 'Jane is more intelligent than Jack.'), both questions have to be answered affirmatively; they imply the existence of multidimensional properties, properties which 'are composed of more elementary properties,' (ibid., p. 54) where the latter are stereotypical for the kinds of things which have the multidimensional property.

Van Woudenberg and Peels provide a positive description of the role such properties play via à vis degree-involving sentences: They are both sensitive to degrees (positive answer to (A)) and admit of a 'more or less' (positive answer to (B)). How many of the aspects which e.g. compose wisdom a person has then determines the degree to which they are wise, and if Socrates has more of them than Diogenes, then it is true (at that time) that he is wiser than Diogenes. (See ibid., §5.) Multidimensional properties hence provide us with truthmakers for certain kinds of degree-involving sentences, but not for all of them. This qualification is important, since as pointed out clearly in van Woudenberg and Peels (2018), not every degree-involving sentence implies the existence of multidimensional properties. The sentence 'Michael scored more goals than John.' for example concerns a comparison between numbers of tokens of the same type, rather than a comparison between numbers of aspects of a multidimensional property. (See ibid., p. 51.)

van Woudenberg and Peels (2018) further flesh out this picture in two important ways. First, they refine it by pointing out that some aspects may be be more central

than others. (See ibid., p. 53.) For example, the scale by which freedom is measured in the context of comparisons arguably has to attach different weights to different aspects of freedom, since freedom from political repression should count more towards being free in general than the freedom afforded by the absence of, say, a duty to wear a school uniform. Second, such scales are sometimes, as van Woudenberg and Peels put it, somewhat opaque: They admit of cases in which the scale fails to determine how two compared items relate to each other. An example is the comparison of two people with respect to two aspects of (negative) freedom, freedom from psychological and from social impediments. (See ibid., p. 58.)

Van Woudenberg and Peels complement this positive characterisation by criticising three potential ways to reductively explain the relation between multidimensional properties and their aspects. (See ibid., p. 53.) First, this relation is not a determinable-determinate relation. Having a single determinate of a determinable implies having the determinable (think e.g. of *green* and *being coloured*), but this is not generally the case for having a single aspect of a multidimensional property. Think e.g. of *mastering basic arithmetic* and *being wise*. Having the former is not always sufficient for having the latter simpliciter, as I have put it above.

Second, it is also not a genus-species relation. A genus can always be conjoined with a specific characteristic to form a species (e.g. the genus *parallelogram* plus the characteristic *having equal sides* gives us the species *rhombus*). In contrast, it is not clear to which genus one would have to add e.g. *being able to take fair decisions* in order to produce *wisdom*.

Third, aspects do not relate to their aspect in the same way as types relate to their tokens: E.g. bodily, financial, or psychological freedom are not tokens of the type freedom, since it is essential to tokens that they can be (with differences only in superficial qualities related to the material of which they are made) repeated (think of Descartes *Meditations* and particular printed books containing its text), whereas it makes no sense to assume that there are multiple, qualitatively (almost) indistinguishable financial freedoms. (See ibid, p. 58.)

Let me expand on this negative part of van Woudenberg and Peels' characterisation by providing arguments against three further reductive accounts which one might be tempted to entertain.

First, we cannot assume that the relation between a multidimensional property and its aspects is analogous to that of a conjunctive sentence to its conjuncts. I.e. multidimensional properties are not conjunctive properties. If they were, then for Socrates to be wise would be for him to be able to solve complex problems *and* to be able to

mediate between conflicting opinions *and* .... In other words, having a multidimensional property would always require having all of its aspects. This will not do, since, as van Woudenberg and Peels point out, a thing can have a multidimensional property (simpliciter), even though it only has a proper subset of its aspects.

Second, aspecthood can also not be taken to be analogous to disjunction. If it were, then having any single arbitrary aspect of a multidimensional property would by itself always be sufficient for having that property (simpliciter). Alas, being wise is not that easy!

For the very same reason, multidimensional properties are also not multiply realizable, in the sense that a multidimensional property's aspects are not just different realizations of that property. Each realization of a property is usually taken to be sufficient for the relevant object's having the multiply realizable property simpliciter, which is, once more, not the case for having aspects of multidimensional properties.

Similarly, multidimensional properties are also not to be confused with structural universals, which require proper parts of the object which instantiate them to have certain distinct properties, or to stand in certain relations. (Cf. Fisher (2018).) Think, for example, of the property of being a water molecule and the properties of being a hydrogen and an oxygen atom, properties which parts of the molecule have to have in order for the whole molecule to instantiate the (assumed) structural universal of being a water molecule. For an object to have a multidimensional property, it has to have at least one of that property's aspects, but there is in general no reason to think that having such a property requires particular relations to obtain between its mereological parts. Again, the proposed metaphysical explanans does not match up to the explanandum.

3 A reduction of multidimensional properties in terms of conjunctive-disjunctive parthood

The preceding discussion urges us towards a non-reductive view of aspecthood: If multidimensional properties are distinct from the discussed range of different kinds of complex properties, then this gives us (defeasible) evidence against Reductionism. I will now argue that a reductive account is available after all.

As previously pointed out, multidimensional properties are neither disjunctive, nor conjunctive properties. The account I want to propose works with a specific combination of both kinds of properties to match the characterization of multidimensional properties given by van Woudenberg and Peels. In a nutshell, the account says that multidimensional properties are disjunctive properties which have at least some conjunc-

tive properties as their disjuncts. Let me call such properties 'conjunctive-disjunctive properties'.

Recall that the problem with the 'purely' disjunctive account is that it would always make having one aspect fully sufficient for having the multidimensional property simpliciter. What van Woudenberg and Peels claim is that in order to have a multidimensional property, having one aspect of the property is sometimes not enough, whereas there will always be certain subsets of aspects of that property such that having all of them is. As they put it: 'There is one thing, the property of being wise, which is composed of properties that are individually neither necessary nor sufficient for being wise, but conjunctions of which are sufficient for it.' (van Woudenberg and Peels (2018), p. 54.) This idea can be captured by the conjunctive-disjunctive Reductionist account. According to it, being wise is a conjunctive-disjunctive property, that is, a property which is disjunctive and has conjunctive properties as its disjuncts. The conjuncts of these conjunctive properties are then its aspects. To apply this account to the example we started with: *Being wise* is taken to be a conjunctive-disjunctive property which has the property *having a rich and diverse body of knowledge*, *being very good at taking fair decision*, and *having a rich and diverse body of knowledge AND being very good at taking fair decision*, plus further conjunctive or non-conjunctive wisdom-conducive properties, as its disjuncts. Sam has only the first of these properties, Max only the second, Kim has both and hence also the conjunctive property, and, in virtue of having these properties, all three of them may have the overarching conjunctive-disjunctive property of *being wise* simpliciter, depending on what the requirements of having it simpliciter are in the relevant context.

I now want to introduce this account in a slightly more detailed and systematic fashion. To do so, I will rely on a notion familiar from the recent metaphysical literature, namely that of quasi-subsumption introduced in Correia and Skiles (2019). This notion, which is defined in terms of disjunctive and conjunctive parthood, which in turn are defined in terms of the primitive notion of generalized identity, can be taken to capture a particular relation between properties which almost exactly resembles the relation which multidimensional properties have to their aspects if the conjunctive-disjunctive property-account is adopted.[6] Only 'almost exactly', because while quasi-subsumption gives us the right metaphysical structure to capture the relation between multidimensional properties and their aspects according to van Woudenberg and Peels' characterisation, the logic of generalized identity suggested in Correia and Skiles (2019) is not a perfect match

[6]Note that Correia and Skiles (2019) are careful not to commit themselves to a realist view of properties like the one assumed in this paper.

for modelling the behaviour of multidimensional properties.[7] If one wants to extend the current account by aligning it with a full logic of generalized identity, one can hence not simply rely on the work done in Correia and Skiles (2019) but has to go beyond it to adapt it to the needs of a logic of multidimensional properties. An advantage of a fuller Reductionist account of this kind would be that it would establish a firm connection between quasi-subsumption and the notion of generalized identity, potentially further illuminating the idea that multidimensional properties are in some sense constituted by their aspects by ultimately spelling out what 'constitution' means in terms of generalized identity. Since I merely rely on their terminology for illustrative purposes, I leave the task of specifying a matching logic of generalized identity aside in this paper.

To introduce Skiles and Correia's terminology, let me start with their basic notion of generalized identity.[8] Generalized identity is, as the name says, a notion which generalizes the standard objectual notion of identity beyond the domain of particulars. It in particular allows one to formulate not only generic identity claims, but also claims about the identity of properties, kinds, and the like. Claims of generalized identity are often expressed by 'just is'-statement, such as 'being human just is being a rational animal.' Using P and Q to symbolize the properties of *being human* and of *being a rational animal* and using '$\equiv_x$' as the operator expressing generalized identity which binds the variable x (see Correia and Skiles (2019), p. 644), we can formalize the claim that for something (i.e. for x) to be human is for it to be a rational animal as: '$Px \equiv_x Qx$.'

The notion of generalized identity can then be used to define the notion of a disjunctive part. Generally, being F is a disjunctive part of being G if, and only if, there exist other properties $H, I, J, \ldots$, such that for something to be G is for it to be F, *or* to be H, *or* to be I, *or* .... Expressing the definiendum (semi-)formally, we get that F is a disjunctive part of G if, and only if, $\exists H, I, \ldots (Gx \equiv_x Fx \vee Hx \vee Ix \vee \ldots)$.

Correia and Skiles use the symbol $\sqsubseteq_x^{\vee}$ to express disjunctive parthood. Hence, we can express that being F is a disjunctive part of being G as $Fx \sqsubseteq_x^{\vee} Gx$. This then allows us define subsumption as the converse of disjunctive parthood: The same formula also

[7] Note that the logic explicitly defined in Correia and Skiles (2019) is a logic of factual identity (strictly speaking a logic of identity between open or closed sentences). As Correia and Skiles (2019), footnote 9, point out, the predicational part of their framework, i.e. the part I rely on here, is compatible with a range of different logics of generalized identity. That said, a straightforward extension of their logic of factual identity would include a number of entailments which seem at odds with the account of multidimensional properties introduced in this section. For example, it would force us to accept that any multidimensional property has itself as an aspect, which is clearly undesirable in the context of a Reductive account of multidimensional properties for metaphysical reasons. I am grateful to an anonymous referee for raising this point.

[8] Besides by Skiles and Correia, the same or similar notions are also discussed in Dorr (2016), Linnebo (2014), and Rayo (2013).

tells us that being G subsumes being F, or equivalently, that being F is subsumed by being G.

Skiles and Correia use subsumption to account for the relation between determinables and their determinates. E.g. being coloured can be taken to subsume being red, being green, and so on. (See Correia and Skiles (2019), p.657.) Since, as van Woudenberg and Peels point out, this relation differs from that between multidimensional properties and their aspects (see the previous section), it comes as no surprise that subsumption on its own does not track the aspecthood relation. Having a single aspect is generally not sufficient for having the corresponding multidimensional property simpliciter, but having a property which is subsumed by another property is, by the definition of subsumption, sufficient for having the latter (subsuming) property (simpliciter).

However, even if one way of having it is not in general sufficient for having a multidimensional property (simpliciter), perhaps having it in *several* ways may be? This is exactly what van Woudenberg and Peels point to when they state that to have a multidimensional property, an object has to have a number of aspects which together form a set which is sufficient for having that multidimensional property. The idea is that in order to be wise, a person typically has to be wise in all of a number of ways which, if had together, are sufficient for being wise simpliciter, at least in a certain context.

This idea can be captured by Skiles and Correia's notion of *quasi-subsumption*. While subsumption is defined in terms of disjunctive parthood, to define quasi-subsumption, we also need conjunctive parthood. Being F is a conjunctive part of being G ($Fx \sqsubseteq_x^{\wedge} Gx$) if, and only if, there are other properties $H, I, \ldots$, such that for something to be G is for it to be *F and H and I and* .... More formally: $\exists H, I, \ldots (Gx \equiv_x Fx \wedge Hx \wedge Ix \wedge \ldots)$

Generally, being G quasi-subsumes being F (in symbols: $Fx \sqsubseteq_x^{\vee\wedge} Gx$) if, and only, if there are some properties *H,I,...*, such that *F and H and I and ...* (i.e. the conjunction of these properties) is subsumed by G. In other words, if F is a conjunctive part of a disjunctive part of G. In symbols: $\exists H, I, \ldots (Fx \wedge Hx \wedge Ix \wedge \ldots \sqsubseteq_x^{\vee} Gx))$.[9]

Applied to wisdom, one can then say that being wise quasi-subsumes being able to solve complex problems, because this property together with a certain set of other properties corresponding to other aspects of being wise, is fully sufficient for being wise simpliciter in a certain context, even though there may also be other (disjunct, or overlapping) sets of properties which together are sufficient for being wise simpliciter in the same context. This exactly matches van Woudenberg and Peels's view that having a multidimensional property is a matter of having a subset of its aspects which is sufficient

[9]Fully spelled out in terms of generalized identity: being G quasi-subsumes being F if, and only if, $\exists H, I, J, \ldots (Gx \equiv_x (Fx \wedge Hx \wedge \ldots) \vee (Ix \wedge Jx \wedge \ldots) \vee \ldots))$.

for having the complex, resultant property (simpliciter).

To put it more generally, aspecthood, as van Woudenberg and Peels conceive of it, can be characterized in terms of quasi-subsumption, which in turn tracks the relation between multidimensional properties conceived of as conjunctive-disjunctive properties and their aspects. So any apparent multidimensional property can instead be taken to be a property which has disjunctive parts which consist of conjunctive parts, where these conjunctive parts correspond to what Primitivists take to be the aspects of that multidimensional property. This gives us a conditional argument for Reductionism, the condition being that we accept conjunctive-disjunctive properties as part of our primitive ideology. If we have that notion, we do not need to accept the existence of a new, sui-generis category of properties in order to account for the truth of certain degree involving sentences in the manner suggested in van Woudenberg and Peels (2018) or more generally, in order to account for the apparently multidimensional nature of certain qualities in a metaphysical framework which posits properties to account for the qualitative similarity of things.

4 Introducing modes of having

The account of multidimensional properties introduced in Calosi and Michels (2025) goes beyond that in van Woudenberg and Peels (2018) by introducing an apparatus of modes of having. The underlying idea is that once we acknowledge that properties like wisdom or health are intrinsically complex, we should go beyond the binary distinction between having and not having. The multidimensional structure of such properties allows us to define a number of further, more fine-grained modes of having, including in particular that of having this property *partly*, *fully*, *to degree d*, and *simpliciter*. In Primitivist terms (that is, directly using a (assumed to be primitive) notion of aspecthood), the four modes of having can be defined in the following way:

Partly having An object x *partly has* the multidimensional property P if, and only if, x has at least one of P's aspects.

Fully having An object x *fully has* the multidimensional property P if, and only if, x has all of P's aspects.

Having to degree *d* An object x *has* the multidimensional property P *to degree* d if, and only if, P has a total number n of aspects, x has m of these aspects, and $d = \frac{m}{n}$.

Having simpliciter An object x *has* the multidimensional property P *simpliciter* if, and only if, x has P to degree $d \geq t$, where t is a contextually determined threshold value.[10]

Calosi and Michels argue that these definitions are useful, since they provide us with metaphysical resources to make sense of the historically and systematically important ideas that certain qualities can be had to intermediate degrees (*having to degree d*) or partly (*partly having*). *Fully having* is the dual of *partly having*, but is also useful in its own way, since it allows one to neatly distinguish between having a 'regular' unidimensional property which can only be had or not had, such as e.g. *is a prime number*, and having a multidimensional property *fully* or to the highest degree. The notion of *having simpliciter* finally gives multidimensional property-theorists a flexible means to accommodate prevalent intuitions about instantiation. In particular, that one may want to attribute e.g. wisdom to someone in a context who does not exhibit every stereotypical trait (which would amount to them *fully having* the property of *being wise*) of a wise person, or to make sense of the apparent fact that in some contexts, having a single such trait (which would however still be sufficient to them *partly having* the property) in this or another context may not suffice for them being wise (simpliciter). The defined notions can in addition potentially back semantic accounts of modifiers like 'somewhat' and '-ish'. (See Calosi and Michels (2025), §2.)

What I want to point out now is that, in order to meet the explanatory demands for accounts of multidimensional properties introduced at the beginning of the paper, the reductive account proposed in the previous section has to be augmented to include modes of having. Recall that such an account should be able to explain i) how things can resemble each other with respect to certain qualities such as wisdom or health, but also how such overall similar things can still differ ii) qualitatively and iii) quantitatively with respect to the relevant quality, and finally, iv) also that things can apparently have qualities of this particular sort without either having them simpliciter in a certain context or without having them to the highest degree. The Reductionist account without modes of having proposed in the previous section has no difficulties meeting i-iii). It explains the overall resemblance between wise people in terms of them all having the same conjunctive-disjunctive property *being wise* and it explains the evident qualitative

[10] These definitions are adapted from Calosi and Michels (2025), §4. Note again that Calosi and Michels do not address the reduction question and, unlike van Woudenberg and Peels, also do not explicitly dismiss particular Reductionist approaches. Note also that HAVING TO DEGREE *D* is more of a proof of concept than a full definition, since it applies only to a particularly simple kind of multidimensional property. See Calosi and Michels (2025), §3.5.3. The same sort of restriction applies to the reductionist definition provided below.

differences between some of them (they are 'wise in different ways') in terms of the possibility of having different (conjunctive or non-conjunctive) properties which are disjuncts of that conjunctive-disjunctive property (e.g. Sam has the property *having a rich and diverse body of knowledge*, Max that of *being very good at taking fair decision*, Kim has the conjunctive property *having a rich and diverse body of knowledge AND being very good at taking fair decision*). It however falls short when it comes to meeting demand iv). This is because the account makes it a necessary precondition for an object's having a multidimensional property that it has a subset of the property's aspects which is such that having all aspects in the set is sufficient for having the property, *independently of the context*. Going back to my initial example, this for example means that it cannot both accommodate one context in which both Sam (who has only one wisdom aspect) and Kim (who has the same, but in conjunction with one additional wisdom aspect) count as wise simpliciter, and another in which only Kim counts as wise simpliciter, though such differing contexts are clearly conceivable. Either Sam's aspect alone is sufficient for being wise simpliciter independently of context (assuming, as proponents of such an account of multidimensional properties likely should, that single properties may count as (trivial) conjunctive properties), or it isn't and only Kim's richer aspect is, also independently of context.

As it stands, the account can hence not fully meet explanatory demand iv). This problem can easily be addressed by an account of multidimensional properties which integrates the specified modes of having, allowing one to clearly distinguish between having a multidimensional property simpliciter in a context, and 'merely' having it partly.

Generally, the four definitions give us a more nuanced picture of what it means to have a multidimensional property. This naturally leads to the question of what 'having the multidimensional property' should be taken to mean in general, i.e. if none of the four defined modes of having is explicitly specified. A plausible answer is that it should more often than not be understood in terms of *having simpliciter* (as already suggested by my use of the notion throughout the paper). Everyday talk about multidimensional properties usually seems to involve a context sensitive standard for what counts as having the relevant property, but not precise degrees. The definition of *having simpliciter* is hence a perfect match, given that it involves a context-relative threshold-value which gives us a sufficiency condition, matching everyday, loose talk about multidimensional properties. Accordingly, van Woudenberg and Peels' idea transposed to the more nuanced picture is that, for any context, aspects of a multidimensional property typically form sets whose combined *degree of having* allows an object which has all aspects in the set to meet the

threshold for having the multidimensional property *simpliciter*. This allows an object to satisfy the definition for *partly having* without satisfying that of *having simpliciter* with respect to a context. This happens when an object has some conjunctive property which is a disjunct of a conjunctive-disjunctive property, but having that conjunctive property does not make the object have the multidimensional property *to a degree* which is high enough for it to have it *simpliciter*.

In order to show that the proposed reduction of multidimensional properties can accommodate the modes of having fixed by the four definitions, one has to show that they can be recast in those Reductionist terms in which the account is cast. In other words, Reductionists have to provide translations of the definitions using the vocabulary of their account, giving them their own set of definitions of modes of having. The idea is that they cannot only avoid having to posit multidimensional properties to do certain metaphysical jobs, such as e.g. figuring in truthmakers for certain degree-involving sentences, but furthermore also use them to meet the fourth explanatory demand for theories of multidimensional properties, and more generally, enable them to make sense of the idea that some qualities admit of gradation (see Calosi and Michels (2025)). Putting it in a different perspective, if Reductionists cannot provide their own definition of modes of having, then a Primitivist could rightly complain that the proposed reduction is incomplete.

This worry is however unfounded, since once can indeed complete the reduction by recasting the four definitions in the vocabulary of the conjunctive-disjunctive properties account. For ease of formulation, I will (keeping the caveat mentioned at the beginning of section 3 in mind) once more rely on Correia and Skiles (2019) notion of quasi-subsumption to state the Reductionists versions of the definitions:

Partly having^R An object x *partly has* the (quasi-subsuming) property P if, and only if, there is a property Q such that P quasi-subsumes Q and x has Q.

Fully having^R An object x *fully has* the (quasi-subsuming) property P if, and only if, for all properties $Q_1, \ldots, Q_n$ which are quasi-subsumed by P, x has them.

Having to degree d^{R} An object x *has* the (quasi-subsuming) property P *to degree* d if, and only if, n is the total number of properties quasi-subsumed by P, x has m of these quasi-subsumed properties, and $d = \frac{m}{n}$.

Having simpliciter^R An object x *has* the (quasi-subsuming) property P *simpliciter* if, and only if, x has P to degree $d \geq t$, where t is a contextually determined threshold

value.[11]

These definitions provide Reductionist with the same inventory of modes of having which Primitivists, who work with their own versions of the definitions, have at their disposal.[12] They allow the four modes of having to play the same theoretical roles as their Primitivist counterparts (some stated above) and they address the problem that the proposed Reductive account cannot meet all the explanatory demands formulated in the beginning of the paper without them. This, together with the commonplace parsimony-argument, that a theory which posits less primitive bits of ideology than another is preferable, gives us a reason to prefer the Reductionist account developed in this paper to a Primitivist one.

5 Reductionism vs Primitivism: What's Next?

In this paper, I have first argued that assuming van Woudenberg and Peels (2018)'s characterisation of multidimensional properties as a starting point, we need not admit them into our ontology as a primitive kind. Instead, we can rely on conjunctive-disjunctive properties to provide a Reductive account. This account identifies the relation between a multidimensional property and its aspects with, using again Correia and Skiles (2019)'s vocabulary, the relation between a property which quasi-subsumes other properties and the properties it quasi-subsumes. As was shown in the previous section, this first stab at a Reductive account is itself not satisfactory, since it does not meet all four explanatory demands set out earlier in the paper. However, the problem can be addressed by extending the account to include analogues to the four different modes of having a multidimensional property introduced in Calosi and Michels (2025).

Some may prefer the Reductionist account for reasons of parsimony, but are there other differences between Reductionism and Primitivism which might be relevant regarding one's choice between the two approaches? In this last section of the paper, I want to draw attention to one, perhaps subtle, but seemingly significant difference between the two sets of definitions of modes of having associated with the two kinds of accounts.

[11] Note that this definition clearly delineates the only role which contexts play in the Reductive account presented here. In principle, one could allocate a further role to contexts by allowing multidimensional properties to have different aspects in different contexts. It would be interesting to consider the consequences of this change and to discuss whether it could provide an alternative approach to meeting the fourth explanatory demand. I will not discuss this further in this paper, but would like to thank an anonymous reviewer for discussion of this point.

[12] As in the case of the Primitivist definitions, we could add further definitions giving us further modes of having, such e.g. a notion of *strictly partly having*, which applies just in case an object *partly*, but not *fully* has a property.

The general thought underlying what follows in this section is that one can distinguish between two senses in which the Reductionist versions of the definitions are, or are not equivalent to their Primitivist counterparts. They are equivalent to them, in the sense that they capture the same ideas and provide us with notion which are apt to play the same theoretical roles as their Primitivist counterparts. In another sense however, the two sets of definitions diverge: They are not necessarily extensionally equivalent.

To clarify what I mean by that, two definitions are *extensionally equivalent* if, and only if, they apply to exactly the same sets of entities. They are *necessarily extensionally equivalent* if, and only if, they are extensionally equivalent in exactly the same set of possible circumstances. What I want to point out now is that there are conceivable circumstances in which the definitions come apart. I will do so by providing an example.

Being able to touch one's nose with one's index finger is usually not taken to be a stereotypical indicator of a person's wisdom. However, thinking in terms of degrees of wisdom, one could make the case that it contributes to an extremely small degree to being overall wise and should hence be considered an aspect of wisdom. After all, it is, for example, not entirely unwise to follow a police officer's request to exercise this ability when one is stopped driving a car on the highway. It then seems that the ability should be among the aspects of being wise.

At the same time, one might think that manifesting this ability may not be part of any set of properties which are together sufficient for being wise. The idea is that the ability might simply not contribute enough towards making someone wise and that accordingly, it should not be quasi-subsumed by being wise. The point is that 'being an aspect of' and 'being quasi-subsumed by' are prima facie distinct metaphysical categories and that there may be no guarantee that they necessarily coincide. If they can indeed diverge, as the example may be taken to suggest, then we would be faced with a scenario in which Primitivism and a Reductionism pull in different directions with respect to whether a property counts as an aspect of a multidimensional property in the first place. Applied to the example, if we assume the Primitivist definitions of the four modes of having, someone who manifests only this particular ability would in this scenario be *partly*, but not *fully* wise, has wisdom *to a degree d* for some d, and may, in a context in which the threshold is low enough, also be *wise simpliciter*. In contrast, the same person would trivially fail to satisfy all of the Reductionist definitions, meaning that a Reductionist cannot even account for them having a (however minuscule) degree of wisdom.

That one can come up with scenarios of this sort is not all too surprising. After all, there is a sense in which the Reductionist versions of the definitions are more demanding than the Primitivist ones. The Primitivist definitions place no overt restrictions on which

properties can be aspects of multidimensional properties. Reductionists in contrast place what appears to be substantive constraint on the properties which can play the role of aspects according to these definitions, namely that they are quasi-subsumed by these properties.

The described scenario may not be one which we are likely to encounter outside the philosophy room, but it appears to be coherent and may not even be too far removed from actuality. Is this difference significant enough to matter when it comes to choosing between a Primitivist and a Reductionist account of multidimensional properties? I pose this question not to answer it, but simply to indicate that there are still open questions about the two kinds of accounts which may be worth addressing in future work on this topic.

Acknowledgements

Material used in or closely related to this paper was discussed at the eidos seminar at the University of Geneva at two occasions and at the research colloquium of the Chair in Theoretical Philosophy at Goethe University Frankfurt am Main. I'm very grateful for all the discussion and the feedback I received at these and other occasions, including from Claudio Calosi, David Chabote, Fabrice Correia, Edgars Darroberst, Florian Gatignon, Martin Glazier, Roman Heil, Bruno Jacinto, Hugo Luzio, Giovanni Merlo, Marta Pedroni, Martin Pleitz, Albert Sole, Lisa Vogt, Daniel Weger, Alexandra Zinke, and from a number of anonymous referees.

My work on this paper was supported by the Fundação para a Ciência e a Tecnologia (FCT) (CEEC IND4ed: `https://doi.org/10.54499/2021.03171.CEECIND/CP1702/CT0015`, PeX-project 'Indeterminacy in Science': `https://doi.org/10.54499/2023.15136.PEX`.

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