Vagueness, Fuzzy Sets and Metaphysical Indeterminacy

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Can the adoption of a non-classical logic incur substantial metaphysical commitments? According to Rosen and Smith, the answer to this question is 'yes': by accepting the fuzzy logic-based theory of linguistic vagueness, one also has to accept the existence of metaphysical indeterminacy. In this paper, I analyse their argument for this claim and identify a loophole. The argument ignores a salient possibility which undermines it, namely that the metaphysical correspondents of fuzzy sets can be, instead of single fuzzy properties, more complex metaphysical structures, namely graded qualities. Friends of the fuzzy-logic-based theory of vagueness can hence stay clear of the commitment to metaphysical indeterminacy.

1 Vagueness as a gateway to metaphysical indeterminacy?

Non-classical logics have been widely applied across a number of different fields. In philosophy, perhaps the most prominent application is in theories of vagueness. An interesting and under-explored question about the adoption of such logics is whether such logics incur special metaphysical commitments.[1] In a seminal paper (Rosen and Smith (2004)) and a monograph (Smith (2008)), Rosen and Smith have proposed a positive answer to this question. In particular, they argue that if one adopts a fuzzy-logic-based theory of vagueness (*fuzzy view* for short), one thereby also has to accept the existence of metaphysical indeterminacy.

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[1] For an overview of the main issue discussed in conjunction with the adoption of logics in the literature, see Fairbank and Hlobil (2025).

That there is vagueness in language is beyond reasonable doubt, but the same cannot be said about metaphysical indeterminacy, indeterminacy which affects the world itself. To be sure, Lewis's famous quip that '[t]he only intelligible account of vagueness locates it in our thought and language' (Lewis (1986), p. 212) is now all but passé; we have several coherent and systematic theories of metaphysical indeterminacy. However, intelligibility is one thing, applicability another. It is still highly controversial whether there is metaphysical indeterminacy, even if we take what seems to be the best case for its actual existence to date, arguments to the effect that quantum mechanics may imply its existence, into account. (See Torza (2023), pp. 76-7.)

If Rosen and Smith are right, then one of the standard theories of vagueness not only has a surprising metaphysical consequence,[2] but is also burdened with a considerable theoretical cost. A striking conclusion, which might in the eyes of some even provide us with a reductio of this theory and which in addition raises general questions about the consequences of applying fuzzy-logic or degrees of truth in the context of philosophical theories.

In this paper, I will rely on recent work on gradation in metaphysics to rebut the argument. My plan is as follows. I will provide a first analysis of Rosen and Smith's argument and introduce an objection which they themselves raise and dismiss, the objection that their argument takes the model theory too seriously (§2). In the next section, I point out that there are two ways to take the model theory seriously without taking it *too* seriously which differ over a particular assumption about the relation between model and world which I dub the one-to-one-assumption. (§3) I argue that this assumption can be given up in the context of the argument and then draw on recent work on metaphysical gradation to show that one premise is unjustified given this reading. I then introduce a second analysis of the argument which does not involve this premise and argue against its conclusiveness by showing that recent metaphysical models for graded qualities can provide metaphysical correspondents for the fuzzy sets at work in the fuzzy theory of vagueness's model theory, but do not carry a commitment to the existence of metaphysical indeterminacy. (§4) In the last section of the paper, I discuss whether Rosen and Smith's appeal to the notion of definiteness in the argument furnishes them

[2]Note that Merricks (2001) and Taylor and Burgess (2015) have provided important rejoinders to the view that the only kind of indeterminacy is linguistic indeterminacy and have furthermore argued that the step from accepting linguistic vagueness to accepting metaphysical indeterminacy may be smaller than it might seem, or even that semantic and metaphysical indeterminacy may be so closely connected as to necessarily coincide (Taylor and Burgess (2015)). I will not engage with their arguments here since Rosen and Smith's argument is sufficiently independent from them and interesting in its own right. Crucially, their argument, if sound, concerns any application of fuzzy logic, not just its application in a theory of vagueness.

with a response to my criticism. (§5)

2 Rosen and Smith (2004)'s argument

After pointing out that the supervaluationist theory of vagueness does not imply that there is metaphysical indeterminacy, Rosen and Smith say the following about fuzzy-logic based theories of vagueness:

> On the standard fuzzy view, on the other hand, predicate vagueness is primarily a metaphysical matter. When I say 'Bob is tall' I pick out a unique property, tallness: there is no semantic indecision here at all (i.e., no vagueness in the relationship between language and the world). Rather, the property I pick out (which corresponds to the fuzzy set of tall things) is such that things can possess it to any degree between 0 and 1. Thus the reason my sentence is neither simply true nor simply false is that the predicate picks out a unique property of which Bob is a borderline instance. It is a matter of how things are in the world that Bob neither definitely possesses nor definitely fails to possess the property tallness. (Rosen and Smith (2004), p. 186.)

What follows will require a decent amount of verbosity, so it is useful to break this argument down into its core steps. Here is a first stab at a restatement:

1. According to the fuzzy view of vagueness, each vague predicate is assigned a unique meaning in the form of a fuzzy set.
2. To each fuzzy set corresponds a unique fuzzy property.
3. This property can be possessed by things to any degree between 0 and 1 (since it mirrors the structure given by a fuzzy set).
4. If an object possesses this property to an intermediate degree, then it neither definitely possesses, nor definitely fails to possess the property.
5. Lack of definiteness of property possession is a metaphysical matter.
6. Therefore, if the fuzzy theory of vagueness is true, there is metaphysical indeterminacy.

So the argument builds on an inference to the existence of indeterminate properties and on the idea that metaphysical indeterminacy arises from a failure of definite instantiation

of such properties. In his book-length defence of the fuzzy-logic based theory, Smith sticks to the conclusion of the argument, but without explicitly relying on premises 4 and 5, i.e. without recourse to indefiniteness of property-possession: 'If the fuzzy view is correct, there exist vague properties and relations alongside the precise ones (indeed, the precise ones are those special cases of the vague ones that map all n-tuples to 0 or 1)—and this is one perfectly genuine sort of worldly or metaphysical vagueness.' (Smith (2008), p. 71; see also Smith (2008), p. 122.) Smith's main justification for this view is his adoption of a stance towards model theory which he calls 'Semantic Realism'. (See Smith (2008), p. 45.) In the following, I will provide arguments both against this abridged, as well as the full version of the argument. I will not discuss Smith's Semantic Realism in its full generality, but the crucial aspect of the view which drives the abridged and full arguments will be in full focus in the next sections.

3 Fuzzy model theory

What does the model theory of the fuzzy-logic based theory of vagueness look like? Following Smith (2008), a fuzzy interpretation of a language is a tuple $\langle M, I\rangle$, consisting of a Domain M, a set of objects, and an interpretation I, a function assigning semantic values to terms of the interpreted language, that is, truth-values to propositional constants, objects to individual constants, and 'an n-ary fuzzy relation *I(P)* on M to every n-ary predicate P.' (Smith (2008), p. 64.) n-ary fuzzy relations are identified with their characteristic functions, i.e. with functions which map sets of tuples of object from M to elements from the closed interval of the real numbers [0,1] (the set of truth-values). The model theory hence differs from a classical model theory for first-order predicate logic in that it involves a larger set of truth-values and in assigning fuzzy, rather than 'ordinary' sets to predicates as their semantic values. Truth relative to an interpretation for closed atomic sentences is then defined based on *I(P)*: the truth-value of a sentence of the form '$P(a_1, \ldots, a_n)$' is given by $I(P)(I(a_1), \ldots, I(a_n))$, i.e. the truth-value returned by the characteristic function of P for the ordered set as input, which contains the objects assigned by I to the constants $a_1, \ldots, a_n$. This characteristic function can be identified with the fuzzy set which captures the meaning of the predicate P, generalizing Zadeh's original definition of the notion (see Zadeh (1965), p. 339) to sets containing ordered sets.

The recursive definition for the truth of logically complex sentences involving the logical connectives and the quantifiers does not matter with respect to Rosen and Smith's argument, so we need not discuss it. The crucial bit is the semantic interpretation of

predicates. Predicates are assigned fuzzy sets, which can respectively be identified with functions from objects from the domain to a truth-value (unary predicates) and tuples of objects from the domain to truth-values (relational predicates). The crucial question now is how these fuzzy sets relate to properties/relations.

First of all, one might think that fuzzy sets already *are* properties, accepting a suitably adapted version of class nominalism (see Armstrong (1989), pp. 8ff) or a similar view. I will put this approach to the argument aside for the moment. If that view were correct, then the point of the argument must either be that the existence of fuzzy sets alone already entails that there is metaphysical indeterminacy (as per Smith's abridged version of the argument), or that metaphysical indeterminacy enters the picture only through the possibility that these sets/properties can be indefinitely possessed by objects (premises 4 and 5 of my reconstruction of the argument). In the former case, the argument would be superfluous, since the issue of the implication to the existence metaphysical indeterminacy would then boil down to the issue of the existence of fuzzy sets, i.e. an issue on which Rosen and Smith's argument has nothing new to say, since it simply presupposes their existence. The latter case is more relevant. I will set it aside for the moment, but come back to it in the last section of the paper.

If we instead take Rosen and Smith by their word that the property a vague predicate refers to 'corresponds' (op. cit.) to a fuzzy set, the argument has to establish the further point that the fuzzy sets which do their work in the model theory correspond to something on the metaphysical level which gives rise to, or is subject to metaphysical indeterminacy. This second approach is based on a three-layer view: We have a language which is subject to semantic analysis (first layer – predicates of a vague language), a model theory which specifies the meanings of expressions and sentences of the language (second layer – the mathematical entities which the semantic theory posits in order to capture (aspects of) the meaning of the predicates from the first level), and the language independent reality about which the language transmits information (third level – whatever entity corresponds to the meaningful predicate in the world, e.g. a property or a relation). For the next two sections, the focus will be on this second approach.

4 Taking the model theory seriously

Rosen and Smith briefly mention an objection to their argument, namely that it 'takes the model theory too seriously'. Relying on my reconstruction of the argument, the target of this objection are premises 1, 2, and 3: The model theory relies on fuzzy sets

as the semantic values of predicates, assigning one such set to each predicate. To take the model theory too seriously in the sense of the objection is to read off the underlying metaphysical picture from these facts about the semantics, i.e. to assume that there is one property corresponding to each predicate and that this property must in some sense mirror what the fuzzy set does in the semantics. That is, it has to be a property which an object can have to an intermediate degree, rather than just have or not have *tertium non datur*), and, according to premises 4 and 5 of my reconstruction, which the object can also fail to definitely possess. While Rosen and Smith do not further elaborate on the nature of such fuzzy properties, it seems natural to think that these kinds of properties must genuinely differ from non-fuzzy properties. Presumably in a way which guarantees that having them can be a matter of genuine metaphysical indeterminacy.

Rosen and Smith's response to the objection is simple. They insist that taking the model theory seriously is the only option. I will grant them this response, but want to resist the idea that taking the model theory seriously *in the way they suggest* is the only option. The core idea of my argument is that Rosen and Smith's argument involves a particular assumption about the relation between fuzzy sets and properties which is dispensable in the context of the fuzzy view.

Based on the three-level view described in the previous section, the mentioned assumption is that there is a one-to-one correspondence between the fuzzy set assigned to a predicate as its meaning in the model theory and the property which corresponds to it (indicated by Rosen and Smith's use of 'unique' and the definite article in the quoted passage). One fuzzy set corresponds to one property. For convenience, let me call this the *one-to-one-assumption*.

To clarify this assumption, we need to be clear about the two kinds of things which, according to it, correspond to each others and on the nature of their correspondence. As pointed out in the previous section, we can, in the context of a fuzzy view, identify a fuzzy set with its characteristic function. This is a function which assigns to any ordered set of object from the models' domain a value from the open interval between 0 and 1, where this value indicates the grade of membership of the ordered set in the fuzzy set.

The correspondents of fuzzy sets on the metaphysical level are, per Rosen and Smith's assumption, properties. This in particular means we can put aside alternative metaphysical accounts of qualities, such as trope-based or nominalistic views. More specific questions about the nature of properties, such as whether they are Platonic or Aristotelian universals, likewise do not matter in the current context.

What definitely does matter is the nature of the relation of correspondence at issue in Rosen and Smith's assumption. It holds between a characteristic function of a fuzzy set

and a property, where the latter 'is such that things can possess it to any degree between 0 and 1.' (Ibid.) This indicates that the relation conserves a particular feature of fuzzy sets, namely their gradedness, in the sense that any property which corresponds to such a set in Rosen and Smith's sense also has to admit of gradation. This conservation-property is crucial for Rosen and Smith's argument. According to a very well-established standard view of properties, properties are *not* gradable.[3] Without the conservation-property, we would expect the correspondence to hold between (characteristic functions of) fuzzy sets and standard properties, i.e. properties which do not come in degrees, presumably leaving us with an ontology free of any indeterminacy.

Rosen and Smith's argument requires the conservation of gradedness in this sense, but, and this is the point of departure of my criticism, it does not require the one-to-one-assumption. The gradability of fuzzy sets can be conserved on the metaphysical level if we relax this assumption and allow their metaphysical correspondents to be complexes involving properties.[4]

Is this an admissible move? Can we assume this weaker relation between semantic model and world and a corresponding weaker understanding of what it means to take the model theory seriously? I will now argue that this option is available. My argument draws on a general view of semantic models according to which they essentially play a functional role and an essentialist view of what it means to take the model theory seriously. I take both of these assumptions to be plausible, but will not be able to fully develop or defend them here. The main argument of this paper is hence (as philosophical arguments tend to be) a conditional argument.

Put in general terms, the purpose of a model in a model theoretic semantics is to capture (as much as possible of) the meaning of the expressions of all sentences of a language in a mathematically precise way.[5] The gist of my following argument is that the inference from a feature of a model to a feature of the world it represents is only admissible for essential features and that the one-to-one-assumption is not among these

[3] Armstrong even goes so far as to claim that we cannot 'make much sense of degrees of instantiation.' (Armstrong (1989), p. 48.)

[4] With a nod to Wittgenstein's view of the relation between language and world in the Tractatus, this move could be compared to a move from an isomorphic to a homomorphic relation between structures: We focus on a correspondence between two structures and relax the requirement that the two structures not only share a certain mathematical structure, but also their cardinality. The analogy however has its clear limits, since the correspondence we are concerned with here is cross-categorical, linking a model, which is a mathematical structure, with an ontological structure. The latter notion is subject to controversy, leaving us with a number of substantial metaphysical questions which would have to be settled before we could begin to discuss whether the analogy is more than a mere similarity.

[5] Note that this claim is metaphysically non-committal in the sense that it does not presuppose a particular metaphysical view of what kind of entities semantic models are.

features.

Model theory stands in the tradition of truth-conditional semantics, which means that it accounts for linguistic meaning by providing us with systematic rules for determining the truth-values of all grammatically well-formed sentences of the language. It implements the Fregean idea that the meaning of every word in a language can be captured by providing truth-conditions for all (grammatically well-formed) sentences of the language in which it appears. Fuzzy model theory is a generalization of the standard, bivalent paradigm of this kind of semantic theory. As pointed out in the previous section, the fundamental difference to that paradigm is that additional truth-values are recognized. This difference of course requires us to take additional decisions about the truth-functional definitions of logical connectives, but the fundamental purpose of a model nonetheless remains the same.

How does a fuzzy model from this tradition fulfil its essential purpose? By providing a systematic way for us to determine the truth-values of all logically complex sentences of a language based on an assignment of semantic values to the non-logical elements of the language and a recursive definition of truth in a model which utilizes this assignment in order to capture the meaning of the logical elements of the language. The part of this semantic mechanism at issue in Rosen and Smith's argument is the assignment of semantic values to predicates, i.e. to a non-logical kind of element of the language. This assignment can effectively make its contribution to a model's fulfilling its essential purpose in different ways. Instead of the characteristic function of a fuzzy set, a model could for example be set up to rely on two functions, one 'classical' interpretation-function I^C, which first assigns a regular, non-fuzzy ordered set to a predicate, whose output is then fed into a second function I^T which maps a sets of objects from the model's domain to fuzzy truth-values relative to the relevant predicate. The recursive definition of truth in a model would have to be adjusted to accommodate this difference in assignment of semantic values to predicates, but both models would perform their essential task of capturing the meaning of all terms of the given language by assigning truth-values to all its grammatically well-formed sentences just the same. The second sort of model would be unnecessarily cumbersome, but it would still play its essential theoretical role as well as the more elegant model. Which kind of model we work with hence seems to be a matter of convenience. Of course, proponents of the fuzzy view work with the first kind of models in practice, but they need not necessarily do so. From the functionalist perspective assumed here, this suggests that it is a contingent, that is, non-essential feature of fuzzy models that they posit one rather than two functions to fulfil their purpose.

According to the essentialist view of what it takes to take the model theory seriously which I assume here, inferences from features of a model to features of the world are only permissible for essential features of the model. To illustrate, consider a different kinds of mathematical model, a model of atmospheric convection based on the three Lorenz equations. If we hold a view of laws of nature according to which they are rather model-independent entities in the world,[6] then there is a substantial question about whether these equations are genuine laws. Now assume that in this scenario, we ponder the question of whether we can infer the number of laws of nature governing atmospheric convection from the number of axioms involved in the model. Could we infer that there are three such laws of nature? Plausibly we couldn't. The Lorenz equations can be rewritten mathematically as a single axiom which we could use to build a different model with that axiom as its only law. Assuming that the essential functional role of the model is to allow us to make adequate predictions and that both a model involving the three equations and one which involves the single rewritten law play this role equally well, then we should not take the number of laws involved in the physical model as a guide regarding how many laws of nature governing this particular kind of physical system there are. Ti is a merely contingent feature of the model that it involves three equations is a contingent feature of the model which should not be taken to be indicative of an underlying ontological structure. The content of the equations, which is conserved in the re-written axiom, on the other hand can be taken to be indicative of 'something in the world', since it is a feature the model needs to have in order to perform its essential theoretical role.

As the example of the second kind of model shows, the assumption of a one-to-one correspondence between fuzzy-sets assigned to predicates as their semantic values in a model and properties in the world latches onto a contingent feature of fuzzy models which the model need not necessarily have to perform its essential theoretical role.[7] By the essentialist view of taking the model theory seriously, given the mentioned assumptions, not a reliable guide to what the world is like. One can accept the fuzzy view without accepting the one-to-one-assumption.

For their argument to work, a feature of the model which needs to be conserved on the metaphysical level is the gradedness of the semantic values it assigns to predicates. This

[6] Two standard theories of laws of nature which fit this description are the DTA-view (see e.g. Armstrong (1983)) and Primitivst views (see e.g. Maudlin (2007)).

[7] Note that the model of course needs to involve *some* mechanism to assign semantic values to predicates, but crucially, the contingency concerns not *that* such a mechanism is put in place, but rather *which particular implementation* of that kind of mechanism that is. The one-to-one-assumption is tied to this latter kind of choice.

gradedness is the relevant source of metaphysical indeterminacy. In the following section, I will argue that this feature can be preserved without the one-to-one assumption. This will then set up my main point, namely that there are models of metaphysical gradedness which do not entail the existence of metaphysical indeterminacy, opening up a way to resist the conclusion of Rosen and Smith's argument.

With respect to the above reconstruction of their argument, the point made in this section is that its second premise, which says that a unique property corresponds to each fuzzy set present on the semantic level, can be contested. To strengthen this point, I want to point out that I am not the only skeptic questioning the one-to-one-assumption. In *the* classic philosophical presentation of the fuzzy view in Machina (1976), Machina briefly discusses its ontological commitments and explicitly rejects the assumption. He frames the discussion in terms of an example, namely the content of Jones belief that *Horatio planted Petunias in the garden yesterday* (i.e. a proposition) which has an intermediate truth value:

> Ontologically, I suppose this means there is not *a relation of planting*, not even a determinable as opposed to a determinate one, which Jones believes Horatio to stand in with respect to petunias. For, if there were such a relation, I should think that Horatio would either stand in it with respect to the petunias, or not, as the case might be. *Rather, what is going on here is that there are many properties and relations having to do with Horatio's actions vis a vis the petunias – all of which are relevant to the truth value of Jones' belief.* And that belief on this occasion is constructed in such a way that these properties do not all add up to something definitely falling within, or without, Jones' concept of petunia planting. (Machina (1976), p. 58; my emphasis)

Just like Rosen and Smith, Machina works with a model theory in which the vague relational predicate 'planted' corresponds to a single fuzzy set (see Machina (1976), p. 68). However, he clearly does not take this feature of the model as an indication that there is a single corresponding (fuzzy) property. Interestingly, his argument draws on the mentioned classical view of properties and relations, according to which entities either stand in relations or don't, *tertium non datur*. Machina assumes that, on the metaphysical level, there is a multitude of such classical, binary relations and properties which all correspond to the fuzzy set posited as the semantic value of a vague predicate in the model, in the sense that they are relevant to the truth of a proposition involving the predicate. There seems to be no reason to think that this view fails to take the (very

same!) model theory seriously.

The view I present in the next section joins Machina in rejecting the one-to-one-assumption, but at least initially departs from the classical view of properties and relations. It in principle admits properties and relations which can be had, or stood in, to an intermediate degree, but it comes in three variants, two of which ultimately conserve the classical view on a deeper level of metaphysical analysis.

5 Taking the model theory seriously without the one-to-one-assumption

In this section, I will argue that if the one-to-one-assumption is given up, taking the model theory seriously no longer necessarily forces us to accept that there are cases in which it is indeterminate whether an object has a property, or indeterminate whether objects stand in a relation, just because we can successfully apply the fuzzy-logic based theory of vagueness to a vague sentence.

5.1 A second reconstruction of the argument

To start, let us look at Rosen and Smith's argument again, this time in a version which does not presuppose the one-to-one-assumption. The assumption was encoded in the second premise of the previous reconstruction of the argument. If we leave out this premise and adjust the argument accordingly, the argument can be restated as follows:

1* According to the fuzzy view of vagueness, each vague predicate is assigned a unique meaning in the form of a fuzzy set.

2* Each fuzzy set has a metaphysical correspondence which structurally mirrors the fuzzy set in that it allows for things to possess the quality modelled by the predicate to any degree between 0 and 1.

3* If an object has a quality to an intermediate degree, then it neither definitely possesses, nor definitely fails to possess the property.

4* Lack of definiteness of property possession is a metaphysical matter.

5* Therefore, if the fuzzy view of vagueness is true, there is metaphysical indeterminacy.

This new restatement is neutral regarding the number of things on the metaphysical level which account for the gradedness which a fuzzy set captures on the semantic level of the model theory. The word 'quality' is used here as an ontologically neutral term which points to whatever accounts for the qualitative nature of a thing (or the qualitative nature of complexes involving several things) on the metaphysical level, be it a property, relation, or complex structure.[8] Note that this second reconstruction is conservative with respect to the first; if there were a bijection between fuzzy sets and fuzzy property after all, it could, given of course that the other premises also turned out to be true, give us a true instance of the argument scheme. This is arguably the better version of the argument, as it no longer involves the one-to-one-assumption, which I have argued to be dispensable in the previous section.

5.2 Graded qualities

To evaluate this second reconstruction of the argument, we need to discuss what it could mean for there to be a metaphysical correspondence to a fuzzy set which does not consist of a single (fuzzy) property (or relation). Answers can be drawn from the recent literature on the metaphysics of gradedness. In particular, I will draw on Calosi and Michels (2025), who have argued for the sensibility of the idea that things can have certain qualities to a degree and have provided three metaphysical models of such qualities.

I want to briefly introduce all three, then argue that at least two of these models are adequate metaphysical correspondents for fuzzy sets, and finally make the point that the resulting metaphysical underpinning of the fuzzy view's semantics is free of metaphysical indeterminacy.

According to Calosi and Michels (2025)'s first model, an object can have a quality to a degree by having a *degreed property*, a complex property which itself involves a degree, but is itself classical, i.e. can only be had or not had by an object, *tertium non datur*. So if someone has the quality of being wise to degree d, they do so because they have

[8] One might argue that this adds an additional sub-level on the third, the metaphysical level relevant to Rosen and Smith's argument. This is however not a step one needs to take. 'Quality' is really just meant to be a placeholder term which is useful to talk neutrally about the metaphysical level before settling for a concrete metaphysical structure. Also note that the choice of 'quality' instead of similar terms like 'feature' has the implication that the metaphysical correspondents of fuzzy sets are not quantities (in the sense discussed e.g. in Wolff (2020)). This is however not a problem, since the focus on vague predicates already puts quantities out of the picture. Paradigmatic examples of quantitative predicates, think of 'velocity' or 'mass', are not usually vague. Maybe there is a discussion to be had about indeterminate quantities, but this leads us into a different territory altogether. Theories of vagueness are first and foremost designed to handle paradigmatic examples of vague predicates like 'is bald' or 'form a heap' and this gives us the context for Rosen and Smith's argument.

the degreed property *is wise to degree d*, where the latter belongs to a family of complex wisdom-properties which contain different degrees capturing the intensity in which an object has the corresponding quality, but are otherwise perfectly classical.

The second model is more conservative in one respect, namely in that it retains the standard idea that every quality corresponds to one property, so that there is just one property of *being wise* instead of the family of properties posited by the first model. It is however revisionary in another respect, namely in that it posits an instantiation relation which ties objects and properties (or relations) together in order for them to form obtaining states of affairs. According to the second model, it is this instantiation relation which, in a *sui generis* way, admits of degrees.

The third and final model they propose relies on a specific category of properties, multidimensional properties (see also Michels (2025)). Multidimensional properties are complex properties which encompass other properties, their aspects, which convey, alone or together with other aspects, qualitatively different ways of having a quality to an object. To illustrate the idea via an example, two persons may both have the multidimensional property of *being creative*, but they may be creative in vastly different ways. One may be musically creative, the other a creative mathematician, where these different ways of being creative are explained in terms of them having (partly overlapping, but largely disjunct) sets of aspects of the multidimensional creativity-property. The degrees to which an object has a quality is then determined by the number of aspects of the corresponding multidimensional property which an object has, as well as by other factors, such as the weights which these aspects carry with respect to having the relevant quality.

A common feature of all three models is that they rely on a multitude of properties or relations in order to account for the gradedness of a particular quality. The first model posits a family of degreed properties where one might be tempted to (naively!) see just one. In case of the second model, which posits a gradable instantiation relation, this multitude involves a property or relation corresponding to the relevant predication and in addition the instantiation relation. The third finally posits complex properties which in some sense involve other properties as their aspects. Note that the third probably perhaps comes the closest to Machina's view, since one could take his example of the relation of planting to be a multidimensional relation involving a number of classical properties and relations as its aspects. The graded instantiation relation posited by the second model is not a classical relation and the qualitatively homogeneous families of degreed properties posited by the first do not match up with the multitude of qualitatively different properties and relations Machina posits.

What still needs to be addressed is whether the structures provided by these models are apt to play the role of metaphysical correspondents of fuzzy sets in the context of the model theory for vague language provided by the fuzzy view of vagueness. There are really two issues here, corresponding to two questions.

First, a question of extensional fit between the set of vague predicates the fuzzy view accounts for, and the set of graded qualities, which these metaphysical models account for. For my argument to work as well as it can, the latter set has to be large enough to contain a metaphysical correspondent for each vague predicate covered by the fuzzy view. The second question concerns the formal fit between the structure provided by the three models and the structure of fuzzy sets. In general, the former has to match up to the fuzzy sets in the model theory, in the sense that the provided metaphysical structure has to capture the information about degrees of having a quality carried by a fuzzy set. I.e. the relevant aspect of the essential functional structure of a fuzzy model has to be conserved.

Regarding the first question, a definite answer is hard to come by since it is a vague matter which predicates are vague (see Sorensen (1985)) and since which graded qualities there are is also at least an open question. However, a good case can be made that there are at least graded qualities which correspond to paradigm cases of vague predicates. Take the predicate 'is tall', for example. There is no doubt that this predicate is gradable. We can use the predicate to formulate grammatically well-formed and semantically truth-apt claims about the comparative tallness of things, about the tallest, and the least tall things, which are important markers of its gradability. Since we are, following Rosen and Smith, taking the model theory seriously, we have to assume that this predicate corresponds to a sort of property which reflects this gradedness.[9] Regarding properties and grades, the main focus in metaphysics has so far been on quantitative properties (see e.g. Wolff (2020)), properties like mass, length, or velocity which represent fundamental measurable quantities present in scientific theories. It should be clear that by using the

[9]Note that their commitment to taking the model theory seriously precludes an otherwise tempting reaction to the idea that vague predicates correspond to properties, namely to posit an error theory of vague claims backed by a metaphysical view which assumes that only a select class of predicates to which vague predicates do not belong correspond to elite ('real', 'natural', 'fundamental', etc.) properties and relations. If one wants to go this way, one has to answer the question of whether non-elite properties and relations exist. If they do, then Rosen and Smith's argument could still, in principle, go through. The metaphysical indeterminacy whose existence it aims to establish would then concern only the realm of the non-elite. An eliminativist view of non-elite properties, according to which they don't exist, would on the other hand beg the question against Rosen and Smith's argument. The fuzzy view would end up only being applicable to a wholly defective language whose predicates all fail to refer, undermining the argument's presupposition that it makes sense to take the model theory seriously in the first place.

vague predicate 'is tall', we do not refer to a property of this kind. Indeed, if we stopped using the predicate 'is tall' and instead relied only on precise height or length predicates like 'has a height of 156 cm', then there would be no vagueness left to be accounted for by a theory like the fuzzy view. This at least gives us some reason to think that it must refer to some other kind of graded metaphysical structure and graded qualities appear to be a, or perhaps even the most natural candidate. Furthermore, predicates corresponding to paradigmatic examples of graded qualities discussed in the literature, such as wisdom, health, freedom, or justice, are also evidently vague (see e.g. Alston (1967)). This all suffices to show that the match between vague predicates and graded qualities is quite substantial.

Can we show that the match is perfect, i.e. that all vague predicates in principle have their corresponding graded qualities? Probably not, but I don't see a pressing reason to require a perfect answer to the first question. If the paradigm cases of vague predicates are covered, then the response to Rosen and Smith's argument offered here is already a valuable asset for philosophers who accept the fuzzy view, but shy away from accepting that there is metaphysical indeterminacy.

Let us now turn to the second question, the question of the formal fit between the metaphysical structure and the structure of fuzzy sets. A fuzzy set is a function from objects to truth values in the closed interval of the real numbers between 0 and 1. With respect to their second model, the model which posits a graded instantiation relation which ties together properties or relations and objects, Calosi and Michels (2025) have already argued that the metaphysical structure the model provides parallels that of fuzzy sets.

The other models also certainly capture part of the structure of fuzzy sets, since they map grades of having between 0 and 1 to objects, but there is a discussion to be had whether the range of this mapping covers the corresponding closed interval of the real numbers, or whether it falls short of that.

In case of the first, the degreed properties-based model, Calosi and Michels (2025) simply assume it as a working hypothesis that this is the case, i.e. they stipulate that given this model, there is an infinitely large family of degrees properties for every graded quality, each member involving a grade corresponding to a number from the open interval in the real numbers between 0 and 1. With respect to the third model, they point out that the model only covers this range partly in case of the very simple kind of multidimensional property they focus on for illustrative purposes, since it provides only a finite number of degrees for the examples they discuss. (Cf. Calosi and Michels (2025), §3.5.) Perhaps this limitation can be overcome by mapping the degrees it provides to subsets of the full

range of degrees covered by fuzzy sets, drawing on the structure provided by Zadeh's original approach to truth-value sets in fuzzy logic (see Smith (2004), §14). To show that Rosen and Smith's argument is inconclusive, it suffices that one of the three models provides adequate correspondents to fuzzy sets, so I will not further discuss this issue with the third model in this context.

If some metaphysical account of graded properties can do the job, then the pieces needed for my rejoinder to the abridged version of the argument apparently endorsed in Smith (2008) are in place. If fuzzy sets in the model theory of the fuzzy view stand for graded qualities (as opposed to fuzzy properties, i.e. single entities which exactly duplicate the structure of fuzzy sets in, presumably, a *sui generis* way), then commitment to this theory of vagueness does not entail commitment to metaphysical indeterminacy. The idea underlying the abridged argument is that the existence of fuzzy properties and relations alone give us a 'perfectly genuine sort of worldly or metaphysical vagueness.' (Smith (2008), p. 71.) But this turns out to be a mistake given that models for graded properties provide us also with models of the metaphysical correspondents of fuzzy sets in semantic models of the fuzzy view. The degreed properties model makes it particularly easy to see that the underlying metaphysics is then free of any properties which by themselves introduce any metaphysical indeterminacy. If that model is adopted, the metaphysical counterpart of a fuzzy set (or rather, its characteristic function) consists of a class of facts involving objects and the relevant degreed properties, such as e.g. wise to degree 0.1, wise to degree 0.2, etc., which they have. For each object the fuzzy set maps to a particular truth value, there is a (possibly or really obtaining) state of affairs involving the object and a degreed property containing the degree corresponding to that truth value, where this degreed property is, *in an absolute, non-graded ways*, had by the object. None of these facts involves any indeterminacy regarding the instantiation of the involved degreed property by the relevant object. There is hence no indeterminacy to be found on the metaphysical level; the only kind of indeterminacy which is still in the picture is the vagueness of predicates on the first layer, the layer of the object language to which fuzzy models are applied to do their job.

6 Metaphysical indeterminacy through lack of definite instantiation?

In the previous section, the target of my criticism was the abridged version of the argument, whose main idea is that the existence of metaphysical correspondents to fuzzy sets alone suffices to establish that there is a sort of metaphysical indeterminacy. The

additional two premises of the full version of the argument could be seen as providing a rejoinder to my criticism. To repeat, they say that:

3* If an object has a quality to an intermediate degree, then it neither definitely possesses, nor definitely fails to possess the property.

4* Lack of definiteness of property possession is a metaphysical matter.

This could be taken to be an explanation of why the existence of metaphysical correspondents to fuzzy properties is supposed to entail metaphysical indeterminacy, along the following lines: pace Smith's apparent latter inclination in his book, the existence of fuzzy properties alone is not enough to lead to metaphysical indeterminacy. Rather, metaphysical indeterminacy arises through a lack of definiteness of the instantiation of such a property. The indeterminacy hence lies not in the mere possibility that an object may have a fuzzy property to an intermediate degree; rather that an object does have a fuzzy property to an intermediate degree gives us a case of indefinite instantiation, where indefinite instantiation is a form of metaphysical indeterminacy. The core idea of the rejoinder is hence that the metaphysical indeterminacy implied by the fuzzy view only enters the picture through a particular feature of the *instantiation* of a property, namely its lack of definiteness.

For this to constitute an adequate explanation of how metaphysical indeterminacy arises, this lack of definiteness must be a metaphysical matter itself, as stated by the premise 4* of the full argument. This second assumption is needed to ensure that the resulting indeterminacy is really indeterminacy in the world, as opposed to vagueness which merely affects a specific word or concept we use to superimpose on the metaphysical phenomenon of instantiation. If the instantiation of a property could be indefinite, where this indefiniteness merely derived from e.g. a necessary limit to what we can know about instantiation (channelling epistemicism about vagueness) or a specific semantic feature of words of our language, but would nonetheless be perfectly precise on a metaphysical level, then the full version of the reconstructed argument would again fall flat.

The last two points I want to make in this paper are, first, that the standard way in which 'definitely' is handled in the fuzzy view does not give us a way to support the last two premises of the full argument. The second is that the same holds for the closest analogue to a notion of definiteness which the discussed models of graded qualities provide. To already state the conclusion of this section, the points made in the previous section remain valid: accepting the fuzzy view does not force one into accepting the

existence of metaphysical indeterminacy, since there is a salient metaphysical picture according to which the states of affairs corresponding to a fuzzy set in the theory's model theory do not involve any metaphysical indeterminacy.

In the literature on vagueness, 'definitely' has usually been treated as a sentence operator which is introduced into the object language and given a semantic interpretation in the meta-language of theories of vagueness. In general, there are two sensible ways to introduce such an operator. First, one might simply tie definiteness to having one of the two extremal truth-values 0 and 1, i.e. to full truth and falsity. This is Smith's preferred approach (see Smith (2008), p. 257), which he implements by introducing a truth-value operator 'T_1'. When prefixed to a well-formed formula of the object language of the theory, this operator gives us the truth value 1 just in case the embedded formula has the same truth-value, and otherwise the truth-value 0. A complementary operator 'T_0' could be introduced in the same manner, providing the fuzzy view with a means to also express claims about definite lack or absence in terms of being maximally false.

Alternatively, one could instead fix a notion of definiteness in terms of a threshold truth-value, which a sentence has to surpass in order to count as definite. This approach complicates matters a bit regarding the relation between definite truth and definite falsity, but the core idea still remains the same: definiteness is accounted for in terms of having a truth-value in a specific target range.

Whatever the merits of this sort of treatment of definiteness in general are, an appeal to it cannot save the full reconstruction of Rosen and Smith's argument. First of all, the approach provides us with a way to express definite truth and falsity of a sentence and not an account of definite and indefinite instantiation of a property. To apply it in the argument, we first have to transfer what it says about truth to instantiation.

The idea would then simply be that to instantiate a fuzzy property (or other metaphysical correspondent of a fuzzy set) indefinitely is to have it neither to the maximal, nor to the minimal degree (or to instantiate it to a degree outside a specific range of degrees). The problem is that this is not an explanation of why the instantiation of a metaphysical correspondent of a fuzzy set is a matter of metaphysical indeterminacy. Rather, the approach seems to merely allow us to move from the fine grained fact that an object instantiates a property to a degree outside of a specific range of values, to the more coarse grained fact that it both fails to definitely instantiate and definitely fails to instantiate the property, where this fact just is the fact that the object fails to instantiate it to certain degrees. So even if definiteness of instantiation and lack thereof are genuinely metaphysical matters, the treatment of 'definitely' in the fuzzy view does not provide the needed explanation. The rejoinder can hence not provide the sort of

explanation which would be needed to rescue the argument.

One might perhaps think that the rejoinder can save the argument for a different reason. The presented criticism of the argument draws on the discussed models of graded qualities. First of all, if these models turned out to be unable to accommodate the appeal to indefinite instantiation in the last two premises, then this would seem to undermine the response to Rosen and Smith on offer in this paper. After all, if the real driving force behind the argument was an implication of cases of indefinite instantiation, but such cases were ruled out a priori by these models, then the response would seem to beg the question against Rosen and Smith.

This first thought is however groundless. All three models of graded qualities can accommodate notions of indefiniteness. Calosi and Michels (2025), §4 define four modes of having, including *fully having*, i.e. having a graded quality to the maximal degree (matching Smith (2008)'s preferred way of handling definiteness within the fuzzy view), and *having simpliciter*, having a graded quality to a degree which exceeds a contextually determined threshold value. These two modes of having directly give us more coarse-grained metaphysical categories for instantiation facts. However, for the same reason as the categories derived from the treatment of 'definitely' in fuzzy logic discussed above, applying these categories does not provide the sort of explanation needed to ensure that instantiating the metaphysical correspondent to a fuzzy set is an indeterminate matter. That someone e.g. fails to *fully* have and *fully* lack the graded quality of being rich, or that they fail to have it *simpliciter* with respect to a certain context does not suddenly make it an indeterminate matter whether the underlying degreed property, assuming once more the second model provided in Calosi and Michels (2025), is had by that person.[10]

To sum up, Rosen and Smith's argument that the fuzzy view implies metaphysical indeterminacy is not conclusive. I have presented two reconstructions of the argument.

[10]Note that the notion of having simpliciter is subject to indeterminacy in the same manner as every other context-dependent notion. Per definition, objects can neither have, nor fail to have a graded quality *simpliciter* if the context is left unspecified. Of course we can make a claim about some object's having some graded quality simpliciter and not explicitly supply a context with our claim, but in that case, our *claim*, i.e. an utterance, would be indeterminate, not the facts about which object has which graded quality simpliciter with respect to which context. Along these lines, Calosi and Michels (2025), p. 20 admit that reliance on the notion of *having simpliciter* may give rise to indeterminacy, but also argue that this indeterminacy is non-metaphysical. In any case, if context dependency does not introduce metaphysical indeterminacy, then there is no problem. If there turned out to be genuinely metaphysical unsettledness e.g. because some contexts, while themselves free of metaphysical indeterminacy, failed to determine a definite threshold value for when an object has a graded quality simpliciter, then the presented response would have to come with the proviso that the definition of *having simpliciter* provided by Calosi and Michels (2025) should not be taken on board, or be replaced by a suitably adjusted alternative which closes this loophole.

The first involves the dispensable one-to-one-assumption. The second does not, leaving open the possibility that more complex metaphysical structures or multitudes of properties may correspond to the fuzzy sets assigned to predicates within a model. This provides friends of the fuzzy view with a way to resist the argument. They can adopt metaphysical models for graded qualities to paint an alternative metaphysical picture. This picture still takes the model theory seriously in that it posits a metaphysical structure which corresponds to the formal structure of fuzzy sets in the sense that both kinds of structures admit gradedness. However, this alternative metaphysical structure neither introduces metaphysical indeterminacy in virtue of its sheer existence, nor via the account of property instantiation and the complementary notions of definite and indefinite instantiation it countenances. Perhaps there are other reasons to assume that the world itself is indeterminate, but a commitment to the correctness of a fuzzy-logic based theory of vagueness, or more generally to a semantics which relies on fuzzy sets to capture (part of) the meaning of certain kinds of predicates need not commit one to that assumption.

More generally speaking, Rosen and Smith's argument draws our attention to a general question about the relation between indeterminacy and infinity: if a predicate, a concept, or a property is so fine-grained as to be capable of making infinitely many distinctions, does this already constitute a form of indeterminacy? The idea pushed in this paper is that this is not generally the case. If that's right, how can the fuzzy view provide an adequate model for the vagueness of predicates in the first place? The answer might be that it puts infinitely discriminating fuzzy sets in relation with predicates which are tolerant in Wright (1975)'s sense, that is, which by their very nature already fail to make relatively coarse-grained distinctions. If this hypothesis holds water, then an interesting further question to investigate would be whether a similar mismatch in discriminatory power, e.g. between properties belonging to different metaphysical levels, could be exploited to argue for the existence of metaphysical indeterminacy after all. In any case, an argument along these lines would have to have a very different structure from the one Rosen and Smith provide.

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